Math 131C Topics in Analysis. Review for Midterm 1

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Review for Midterm 1, Monday, April 18.

The exam will cover sections 1.1 to 1.3 in the book (Stein-Shakarchi).

Make sure that you know the statements of the main results. You should also know the definitions precisely. There are several main definitions in these sections (exterior measure, measurable subsets of $\mathbf{R}^d$, Lebesgue measure) and quite a few related definitions. For example, a question might ask you to define what it means for a set $E \subset \mathbf{R}^d$ to have measure zero. Your answer should be some variant of:

A set $E \subset \mathbf{R}^d$ has measure zero if, for every $\epsilon > 0$, there is a countable sequence of cubes $Q_1, Q_2, \ldots$ in $\mathbf{R}^d$ such that E is contained in $\cup_{j=1}^{\infty} Q_j$ and $\sum_{j=1}^{\infty} |Q_j| < \epsilon$, where $|Q_j|$ is the volume of the cube Q_j.

There may be "prove or give a counterexample" questions on the exam. To see the sort of proof expected, make sure you understand all the homework problems in homework sets 1 and 2. At least one problem will be taken from the homework, possibly with minor variations.

You should know about infima, suprema, and limits in the real numbers; unions, intersections, and complements of sets; countability, open and closed sets in $\mathbf{R}^d$, compact sets, cubes and rectangles, the Cantor set, the exterior measure on subsets of $\mathbf{R}^d$, Lebesgue measurable sets, the Lebesgue measure, and Borel sets including G_δ and F_σ sets. The homework and the book include some explicit examples that are helpful to know about.

You will probably not be asked to reproduce the proof of one of the main theorems in the book, since that would take too long, in most cases. On the other hand, it should be helpful to understand those proofs as well as you can. The most important thing is to be able to use the theorems for your own proofs.