Homework 6 for Math 131BH Honors Analysis

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Due on Tuesday, March 7.

Rudin, p. 239: 7, 8, 9, 13, 14.

(1) Let $f(x) = x$ for $x \in [-\pi, \pi]$. Compute the Fourier series of f. (If you encounter numbers like $e^{n\pi i}$ or $\cos(n\pi)$ for an integer n, please write them in simpler terms. Also, make sure that your formulas do not involve dividing by 0.)

Show that the Fourier series of f converges (to some value) at every real number x. (Hint: You can use the theorem we proved on pointwise convergence of Fourier series if $x \in (\pi, \pi)$, but that result does not apply if $x = \pi$; why not? So you have to check convergence by hand if $x = \pi$.) Graph the sum of the Fourier series, as a periodic function on $\mathbf{R}$. At which real numbers x does the series converge absolutely?

By evaluating the Fourier series of f at $x = \pi/2$, find an explicit formula for π as an infinite series.