Homework 5 for Math 131BH Honors Analysis

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Due on Tuesday, February 21.

Rudin, p. 196: 6, 12, 19.

(1) Let

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0. \end{cases}$$

Draw the graph of f. Show that f is C^∞ on $\mathbf{R}$.

(2) Using the definition of $\log x$ for $x > 0$ as $\log x = \int_1^x dt/t$, prove that $\log(xy) = \log(x) + \log(y)$ for all $x, y > 0$.

This is a key reason why logarithms were important in scientific computation, from their formal definition by Napier in 1614 until the mid-20th century: once you have a printed table of logarithms ($\log(2.3178) \doteq 0.84062$, $\log(2.3179) \doteq 0.84066$, and so on), you can multiply real numbers quickly and with good accuracy, because the logarithm turns multiplication into addition. Explain (briefly) why this makes more sense than just having a printed table of products.

The slide rule is a simple machine for fast multiplication, based on the same principle.

(3) Using the definition of $\log x$ for $x > 0$ as $\log x = \int_1^x dt/t$, show that $\log x$ is analytic, given by a power series centered around $x = 1$ with radius of convergence 1. Compute that power series.