Homework 3 for Math 131BH Honors Analysis

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Due on Tuesday, February 7.

Rudin, p. 165: 3, 6, 9 (in which E denotes a metric space).

(1) Let (X, d_X) be a nonempty compact metric space. Let $C(X)$ be the set of continuous real-valued functions on X. It follows from results in class that $C(X)$ is a complete metric space if we define distance via the sup norm:

$$d(f, g) = \sup_{x \in X} |f(x) - g(x)|.$$

Show that $C(X)$ is connected but not compact.

(2) Consider the subset of $C([0, 1])$ defined by

$$S = \{f \in C([0, 1]) : f(0) = 0,\ |f(x) - f(y)| \leq |x - y| \text{ for all } x, y \in [0, 1]\}.$$

Prove that S is compact.