

Homework 5 for Math 131AH Honors Analysis

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Due on Tuesday, November 1.

Rudin, p. 44 (ch. 2): 20, 25. P. 78 (ch. 3): 4, 7, 8.

(1) Define a metric space

$$X = \{a = (a_0, a_1, \dots) \in \mathbf{R}^{\mathbf{N}} : \sum_{k=0}^{\infty} |a_k|^2 \leq 1\},$$

with the metric $d(a, b) = [\sum_{k=0}^{\infty} |a_k - b_k|^2]^{1/2}$.

(a) Show that X is a metric space.

(b) Show that X is complete and bounded but not compact. You may use the fact that \mathbf{R}^k is complete for each $k \in \mathbf{N}$.