Homework 4 for Math 131AH Honors Analysis

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Due on Tuesday, October 25.

Rudin, p. 44 (ch. 2): 12, 13, 16, 17, 18.

(1) Show that any open subset of $\mathbf{R}$ can be written as a disjoint union of countably many open intervals.

(2) Let E be a nonempty subset of $\mathbf{R}$ that is both open and closed. Show that $E = \mathbf{R}$.

(3) Let X be the space of polynomials

$$X = \left\{ P(x) = \sum_{k=0}^{n} a_k x^k : n \in \mathbf{Z}^+ \text{ and } a_k \in \mathbf{R} \right\}$$

with the metric

$$d(P,Q) := \left(\sum_{k=0}^{m} |a_k - b_k|^2 \right)^{1/2}$$

for two polynomials $P(x) = \sum_{k=0}^{n} a_k x^k$ and $Q(x) = \sum_{k=0}^{m} b_k x^k$, with $m \geq n$; we define $a_k = 0$ for k greater than the degree of $P(x)$.

(a) Show that d is a metric on X.

(b) Let 0 be the constant zero polynomial. Show that the unit ball $B = \{P \in X : d(P, 0) \leq 1\}$ is closed and bounded but not compact.