Solutions to Exercise Set 5.

1 (a) The noncentrality parameter is $4n(0.15-1/6)^2/(1/6)+2n(0.2-1/6)^2/(1/6)$, which for $n=150$ is $\lambda=1+2=3$. The power, interpolating in the Fix Tables for $\alpha=.05$ and 5 degrees of freedom is only .223. The 5% cut-off point for the χ^2_5 distribution is 11.07. Using the noncentral chi-square tables on the web, we find that the exact probability that a $\chi^2_5(3)$ variable is greater than 11.07 to be .2224. Very low power.

(b) We need $3n/150=16.47$ (from the Fix tables at power = .9). That is, we need $n=824$ observations, to get power .9.

11.1 The $Y_1, Y_2, \ldots$ form an m-dependent stationary sequence with mean $\mathrm{E}Y_1 = \mu_x[\beta_0+\cdots+\beta_m]=\mu_x B$, where $B=\sum_0^m \beta_i$. The covariances are $\sigma_{0t}=\mathrm{Cov}(Y_1,Y_{t+1}) = \sigma_x^2(\beta_0\beta_t+\beta_1\beta_{t+1}+\cdots+\beta_{m-t}\beta_m)$ for $t=0,\ldots,m$. Therefore, $\sqrt{n}(\overline{Y}_n-\mu_x B)\xrightarrow{\mathcal{L}}\mathcal{N}(0,\sigma^2)$, where $\sigma^2=\sigma_{00}+2[\sigma_{01}+\cdots+\sigma_{0m}]=\sigma_x^2B^2$.

For $B=0$, the asymptotic variance is zero so we must rescale to get a nondegenerate distribution. However in this case for large n, the coefficient of the central X_i's in $S_n=\sum_1^n Y_j$, being B, is zero. In fact for $n>m$, $S_n=\sum_{t=1}^m X_t(\sum_{j=0}^{t-1}\beta_j)+\sum_{t=1}^m X_{n+t}(\sum_{j=t}^m\beta_j)$. For $n\ge m$, the distribution of S_n does not change, so S_n converges in law to the distribution of S_m.

6. (a) $\mathrm{E}(Y_n)=2p^2+(1-p)p=p+p^2$. $\mathrm{E}(Y_n^2)=4p^2+(1-p)p=p+3p^2$, so $\mathrm{Var}(Y_n)=(p+3p^2)-(p+p^2)^2=p+2p^2-2p^3-p^4$,

(b) The sequence Y_n is 1-dependent stationary. Since $\mathrm{E}(Y_1Y_2)=4p^3+2(1-p)p^2=2p^2+2p^3$, we have $\mathrm{Cov}(Y_1,Y_2)=(2p^2+2p^3)-(p+p^2)^2=p^2-p^4$. Therefore,

$$\sqrt{n}(\frac{1}{n}S_n-(p+p^2))\xrightarrow{\mathcal{L}}\mathcal{N}(0,\mathrm{Var}(Y_1)+2\mathrm{Cov}(Y_1,Y_2))=\mathcal{N}(0,p+4p^2-2p^3-3p^4).$$