Large Sample Theory
Ferguson

Exercises, Section 8, The Sample Correlation Coefficient.

1. Let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ be a sample from a bivariate distribution with finite fourth moments.

(a) Find the asymptotic distribution of the ratio of the sample variances, s_x^2/s_y^2, as an estimate of σ_x^2/σ_y^2.

(b) Specialize to the case of bivariate normal distributions.

(c) What happens in part (b) as $\rho \to \pm 1$?

2. Let $(X_1, Y_1), \ldots, (X_n, Y_n)$ be a sample of size n from a distribution on the plane with finite fourth moments.

(a) Find the asymptotic distribution of $Z_n = \log(s_x^2/s_y^2)$.

(b) Find an asymptotically distribution-free confidence interval for $\theta = \log(\sigma_x^2/\sigma_y^2)$.

3. Find the variance-stabilizing transformations for $\overline{X}_n$ when sampling from

(a) the gamma distribution, $\mathcal{G}(\alpha, 1)$, with probability density $f(x|\alpha) = x^{\alpha-1}e^{-x}/\Gamma(\alpha)$ on the interval $(0, \infty)$,

(b) the geometric distribution, $\mathrm{P}_\theta(X = x) = (1-\theta)\theta^{x-1}$ for $x = 1, 2, \ldots$.