T. Liggett **Mathematics 31B Final Exam** **December 15, 2004**

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Note: You will find some formulas on the last page that you may find useful.

(15) 1. Define the following:

(a) The series $\sum_n a_n$ converges.

(b) The series $\sum_n a_n$ converges absolutely.

(c) The series $\sum_n a_n$ converges conditionally.

(10) 2. State the integral test for convergence of series.

(10) 3. Compute

(a)
$$\lim_{x\to 2} \frac{\sqrt{2x}-2}{\log(x-1)}$$

(b)
$$\lim_{x\to 1} \frac{x^4-4x^3+8x-5}{x^3-3x+2}$$

(20) 4. For each of the series below, determine whether it converges absolutely, converges conditionally, or diverges. Explain briefly.

	Answer	Reason
(a) $\sum_{n=2}^{\infty} \frac{(-1)^n\sqrt{n}}{\log n}$		
(b) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^n}$		
(c) $\sum_{n=1}^{\infty} \frac{\sqrt{n}\log n}{n^2+2}$		
(d) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^3+50}}$		
(e) $\sum_{n=1}^{\infty} \frac{(-1)^n}{(\log n)^2}$		

(15) 5. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by

$$y = e^x, y = e^{-x}, x = 1$$

about the y-axis.

(10) 6. Find the value of c such that the area of the region enclosed by the parabolas $y = x^2 - c^2$ and $y = c^2 - x^2$ is 576.

(15) 7. Find the volume common to two spheres, each with radius r, if the center of each sphere lies on the surface of the other sphere.

(10) 8. Evaluate the following integrals:

(a) $\displaystyle\int \frac{\cos x}{2+\sin x}dx$

(b) $\displaystyle\int_0^2 x^3\sqrt{x^2+4}dx$

(15) 9. Find the area of the surface obtained by rotating the curve

$$9x = y^2 + 18, \quad 2 \leq x \leq 6,$$

about the x-axis.

(10) 10. (a) Use properties of the logarithm to expand $\ln \sqrt{a(b^2 + c^2)}$.

(b) Solve the equation $2 \ln x = \ln 2 + \ln(3x - 4)$ for x.

(15) 11. (a) Find the radius of convergence and the interval of convergence for each of the two series

$$\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{\sqrt{n}}$$

$$\sum_{n=1}^{\infty} \frac{n(x-4)^n}{n^3+1}$$

(b) A function f is defined by

$$f(x) = 1 + 2x + x^2 + 2x^3 + x^4 + 2x^5 + x^6 + \cdots .$$

Find the interval of convergence of the series, and find an explicit formula for $f(x)$.

(10) 12. By differentiating the geometric series $\sum_{n=0}^{\infty} x^n$, find the following sums explicitly:

(a) $\displaystyle\sum_{n=0}^{\infty} nx^n$

(b) $\displaystyle\sum_{n=0}^{\infty} n^2 x^n$

(15) 13. (a) Find the Taylor series for $f(x) = 1/\sqrt{x}$ centered at $a = 9$.

(b) Express the following indefinite integral as an infinite series:

$$\int \frac{e^x - 1}{x} dx$$

(15) 14. Suppose f and g are one-to-one and twice differentiable functions that are inverses of each other.
(a) Express $g''(x)$ in terms of $f'(g(x))$ and $f''(g(x))$.

(b) Suppose f is decreasing and concave upward. What can you conclude about the concavity of g?

(15) 15. (a) State the root test for convergence of series.

(b) Prove the root test for convergence of series.

Some Formulas

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}, \qquad \cos^2\theta = \frac{1+\cos(2\theta)}{2}, \qquad \sin\theta\cos\theta = \frac{\sin(2\theta)}{2},$$

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}, \qquad \frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}, \qquad \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2},$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \qquad \sec\theta = \frac{1}{\cos\theta}, \qquad \sec^2\theta = \tan^2\theta + 1.$$