Solutions 1

(15) 1. (a) Find the average value of the function $f(x) = x^2\sqrt{1+x^3}$ on the interval [0,2].

$$\text{Average value} = \frac{1}{2}\int_0^2 x^2\sqrt{1+x^3}dx = \frac{1}{6}\int_1^9 \sqrt{u}du = \frac{26}{9}.$$

(b) Give a precise statement of the Mean Value Theorem for Integrals.
See page 403 of the text.

(c) If g is continuous and $\int_1^3 g(x)dx = 8$, explain why g must take the value 4 at least once in the interval [1,3].

By the Mean Value Theorem for Integrals, there is a c in $[1,3]$ so that $8 = \int_1^3 g(x)dx = 2g(c)$. For this c, $g(c) = 4$.

(15) 2. Find the number a so that the line $x = a$ bisects the area under the curve $y = 1/x^2, 1 \le x \le 4$.

The area corresponding to $1 \le x \le a$ is

$$\int_1^a 1/x^2 dx = \frac{a-1}{a}.$$

Therefore the a we want is the solution of $(a-1)/a = (1/2)(3/4)$, or $a = 8/5$.
(15) 3. Use slices to find the volume of the solid obtained by rotating the region bounded by $y = x$ and $y = \sqrt{x}$ about the line $x = 2$.

$$\text{Volume} = \pi\int_0^1 \left((2-y^2)^2 - (2-y)^2\right)dy = \frac{8}{15}\pi.$$

(15) 4. Use cylindrical shells to compute the volume of the solid generated by rotating the region bounded by $y = x^2, y = 0, x = 1, x = 2$ about the line $x = 1$.

$$2\pi\int_1^2 (x-1)x^2 dx = \frac{17}{6}\pi.$$

(15) 5. (a) Find a formula for the inverse of the function

$$f(x) = \frac{4x-1}{2x+3}.$$

Solving

$$y = \frac{4x-1}{2x+3}$$

for x gives

$$x = f^{-1}(y) = \frac{1+3y}{4-2y}.$$

(b) Suppose g is the inverse function of f, and $f(4) = 5, f'(4) = 2/3$. Find $g'(5)$.

$$g'(5) = \frac{1}{f'(g(5))} = \frac{1}{f'(4)} = \frac{3}{2}.$$

(10) 6. Find the equation of the tangent line to $y = e^x/x$ at the point $(1, e)$.

$$y' = \frac{e^x(x-1)}{x^2},$$

which is 0 at $x = 1$. Therefore, the tangent line has equation $y = e$.

(15) 7. Approximate $\int_1^5 x^2 dx$ using $n = 2$ in

(a) the midpoint rule.

$$2(4 + 16) = 40.$$

(b) the trapezoidal rule.

$$1 + 18 + 25 = 44.$$

(c) Simpson's rule.

$$\frac{2}{3}(1 + 36 + 25) = \frac{124}{3}.$$

Solutions 2

(15) 1. (a) Find the average value of the function $f(t) = t\sqrt{1+t^2}$ on the interval [0,5].

$$\text{Average value} = \frac{1}{5}\int_0^5 t\sqrt{1+t^2}dt = \frac{1}{10}\int_1^{26} \sqrt{u}du = \frac{26^{3/2} - 1}{15}.$$

(b) Give a precise statement of the Mean Value Theorem for Integrals.
See page 403 of the text.

(c) If g is continuous and $\int_1^3 g(x)dx = 8$, explain why g must take the value 4 at least once in the interval [1,3].

By the Mean Value Theorem for Integrals, there is a c in $[1, 3]$ so that $8 = \int_1^3 g(x)dx = 2g(c)$. For this c, $g(c) = 4$.

(15) 2. Find the number b so that the line $y = b$ bisects the area under the curve $y = 1/x^2, 1 \le x \le 4$.

The area corresponding to $b \le y \le 1$ (if $\frac{1}{16} < b < 1$) is

$$\int_b^1 \left(\frac{1}{\sqrt{y}} - 1\right) dy = 1 - 2\sqrt{b} + b = (1 - \sqrt{b})^2.$$

Therefore the b we want is the solution of $(1-\sqrt{b})^2 = (1/2)(3/4)$, or $b = (1-\sqrt{3/8})^2$.

(15) 3. Use slices to find the volume of the solid obtained by rotating the region bounded by $y = x$ and $y = \sqrt{x}$ about the line $x = -1$.

$$\text{Volume} = \pi \int_0^1 \big((1+y)^2 - (1+y^2)^2\big)dy = \frac{7}{15}\pi.$$

(15) 4. Use cylindrical shells to compute the volume of the solid generated by rotating the region bounded by $y = x^2, y = 0, x = 1, x = 2$ about the line $x = 2$.

$$2\pi \int_1^2 (2-x)x^2 dx = \frac{11}{6}\pi.$$

(15) 5. (a) Find a formula for the inverse of the function

$$f(x) = \frac{4x-1}{3x+2}.$$

Solving

$$y = \frac{4x-1}{3x+2}$$

for x gives

$$x = f^{-1}(y) = \frac{1+2y}{4-3y}.$$

(b) Suppose g is the inverse function of f, and $f(4) = 5, f'(4) = 3/2$. Find $g'(5)$.

$$g'(5) = \frac{1}{f'(g(5))} = \frac{1}{f'(4)} = \frac{2}{3}.$$

(10) 6. Find the equation of the tangent line to $y = e^x/x$ at the point $(1, e)$.

$$y' = \frac{e^x(x-1)}{x^2},$$

which is 0 at $x = 1$. Therefore, the tangent line has equation $y = e$.

(15) 7. Approximate $\int_0^4 x^2 dx$ using $n = 2$ in

(a) the midpoint rule.

$$2(1+9) = 20.$$

(b) the trapezoidal rule.

$$0 + 8 + 16 = 24.$$

(c) Simpson's rule.

$$\frac{2}{3}(0+16+16) = \frac{64}{3}.$$