

Mathematics 31B/4 – Fall 2004 – Quiz Solutions

Quiz for Tuesday, October 26

Find the inverse function to

$$f(x) = \frac{1 + e^x}{1 - e^x}.$$

Solution: Write

$$y = \frac{1 + e^x}{1 - e^x}$$

and solve for x :

$$e^x = \frac{y - 1}{y + 1}, \quad \text{and then} \quad x = \ln \left(\frac{y - 1}{y + 1} \right).$$

Therefore,

$$f^{-1}(y) = \ln \left(\frac{y - 1}{y + 1} \right).$$

Note, f has domain $(-\infty, 0) \cup (0, \infty)$ and range $(-\infty, -1) \cup (1, \infty)$ so f^{-1} has domain $(-\infty, -1) \cup (1, \infty)$ and range $(-\infty, 0) \cup (0, \infty)$.

2. Find the intervals of concavity (up and down) and the inflection point(s) of

$$f(x) = \frac{\ln x}{\sqrt{x}}.$$

Solution:

$$f'(x) = \frac{2 - \ln x}{2x^{3/2}} \quad \text{and} \quad f''(x) = \frac{3 \ln x - 8}{4x^{5/2}},$$

so the graph of f is concave down on $(0, e^{8/3})$, concave up on $(e^{8/3}, \infty)$, and has an inflection point at $x = e^{8/3}$.

Quiz for Thursday, October 28

1. Evaluate

$$\int_4^9 \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx.$$

Solution:

$$\int_4^9 \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx = \int_4^9 (x + 2 + x^{-1}) dx = \left. \frac{x^2}{2} + 2x + \ln x \right|_4^9 = \frac{85}{2} + \ln \frac{9}{4}.$$

2. Differentiate the following function, and simplify your answer if possible:

$$f(x) = x \cos^{-1} x - \sqrt{1 - x^2}.$$

Solution:

$$f'(x) = -\frac{x}{\sqrt{1 - x^2}} + \cos^{-1} x - \frac{1}{2}(1 - x^2)^{-1/2}(-2x) = \cos^{-1} x.$$