

Mathematics 31B/4 – Fall 2004 – Quiz Solutions

(Note: The sketches that were required are not given below.)

Quiz for Thursday, October 7

1. Use substitution to evaluate

$$\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}}.$$

Solution: Let $u = 1 + 2x$, $du = 2dx$. Then

$$\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}} = \frac{1}{2} \int_1^{27} u^{-\frac{2}{3}} du = \frac{3}{2} u^{\frac{1}{3}} \Big|_1^{27} = 3.$$

2. Sketch the region between the curves $y = 12 - x^2$ and $y = x^2 - 6$, and find the area of this region.

Solution: The two parabolas meet at $(\pm 3, 3)$. Therefore,

$$\text{Area} = \int_{-3}^3 (18 - 2x^2) dx = 18x - \frac{2}{3}x^3 \Big|_{-3}^3 = 72.$$

Quiz for Thursday, October 12

1. Sketch and find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(1, 1)$, and the x -axis.

Solution: The equation of the tangent line is $y = 2x - 1$. Therefore,

$$\text{Area} = \int_0^{\frac{1}{2}} x^2 dx + \int_{\frac{1}{2}}^1 (x-1)^2 dx = \frac{1}{12}.$$

Alternatively, one can integrate with respect to y and end up with a single integral:

$$\text{Area} = \int_0^1 \left(\frac{y+1}{2} - \sqrt{y} \right) dy = \frac{1}{12}.$$

2. Use slices to find the volume of the solid obtained by rotating the region bounded by $y = x$ and $y = \sqrt{x}$ about the line $y = 1$. Be sure to sketch the region.

Solution:

$$\text{Volume} = \pi \int_0^1 \left[(1-x)^2 - (1-\sqrt{x})^2 \right] dx = \frac{\pi}{6}.$$

Quiz for Thursday, October 14

1. Use slices to find the volume of the solid obtained by rotating the region bounded by $y = x$, $y = 0$, $x = 2$, and $x = 4$ about the line $x = 1$. Be sure to sketch the region.

Solution:

$$\text{Volume} = \pi \int_0^2 (9-1) dy + \pi \int_2^4 (9-(y-1)^2) dy = 25\frac{1}{3}\pi.$$

2. Use shells to find the volume of the solid obtained by rotating the region bounded by $x = 4y^2 - y^3$ and $x = 0$ about the x -axis. Be sure to sketch the region.

Solution:

$$\text{Volume} = 2\pi \int_0^4 y(4y^2 - y^3) dy = \frac{512}{5}\pi.$$