Mathematics 275C – Spring 2008 – HW #3

Due: Wednesday, May 14.

1. Let $(B_1(t), B_2(t))$ be standard Brownian motion starting at $(1,0)$. In other words, $B_1(t)$ and $B_2(t)$ are independent Brownian motions starting at 1 and 0 respectively. Fix $\alpha > 0$, and let

$$\tau = \inf\{t > 0 : \alpha B_1(t) = B_2^2(t)\}.$$

(a) Show that $E\tau < \infty$ and compute its value.
(b) Find the means and variances of $B_1(\tau)$ and $B_2(\tau)$.

2. Let

$$X_t = \frac{e^{B_t^2/(1+2t)}}{\sqrt{1+2t}},$$

where B_t is Brownian motion. Use the Markov property to show that X_t is a martingale.

3. Use the result of problem 2 to show that

$$\limsup_{t\to\infty} \frac{|B_t|}{\sqrt{t \log t}} \le 1 \quad a.s.$$

4. Suppose S_n is one dimensional symmetric simple random walk, and let

$$R_n = 1 + \max_{m\le n} S_m - \min_{m\le n} S_m$$

be the number of points visited by time n. Use Donsker's Theorem to show that $R_n/\sqrt{n}$ has a limiting distribution, and write the limiting distribution in terms of Brownian motion.