Mathematics 275C – Spring 2008 – HW #2

Due: Wednesday, April 30.

1. Let $\tau = \inf\{t > 0 : B(t) = 0\}$ be the hitting time of 0, and let $\rho = \inf\{t > 1 : B(t) = 0\}$ and $\lambda = \sup\{t < 1 : B(t) = 0\}$ be the first time 0 is hit after time 1 and the last time 0 is hit before time 1 respectively. Use the Markov property to express the distributions of ρ and λ in terms of the distribution of τ as follows:

(a)
$$P^x(\rho > 1 + t) = \int_{-\infty}^{\infty} p_1(x, y) P^y(\tau > t) dy,$$

and

(b)
$$P^0(\lambda \leq t) = \int_{-\infty}^{\infty} p_t(0, y) P^y(\tau > 1 - t) dy,$$

where as usual, $p_t(x, \cdot)$ is the density of the $N(x, t)$ distribution. You should say explicitly what the bounded measurable function Y is in the application of the Markov property.

2. Show that if σ and τ are stopping times, then so are $\min(\sigma, \tau)$ and $\max(\sigma, \tau)$.

3. Suppose τ is a stopping time, and put

$$\tau_n = \frac{k+1}{2^n} \text{ if } \frac{k}{2^n} \leq \tau < \frac{k+1}{2^n}.$$

Show that τ_n is a stopping time.

4. Let $\tau = \inf\{t > 1 : B_t = 0\}$. Show that τ has density

$$f(s) = \frac{1}{\pi s \sqrt{s-1}}, \quad s > 1.$$