Due: Wednesday, April 18. Note: Problems II and III can be done now; the problems in Durrett can be done after the lecture on Friday, April 14.

I. Problems 2.1 and 2.2 in Durrett.

II. Suppose \( \{ X(t), t \geq 0 \} \) is a stochastic process with stationary and independent increments, and that \( X(t) \) has a symmetric stable distribution with index \( \alpha \in (0, 2] \). Specifically, the characteristic function of \( X(t) \) is
\[
E e^{iuX(t)} = e^{-t|u|^\alpha}.
\]
Note that if \( \alpha = 2 \), then \( X(t) \) is Brownian motion. For general \( \alpha \in (0, 2] \), \( X(t) \) is called a stable process. If \( \alpha < 2 \) the distribution of \( X(1) \) satisfies \( P(|X(1)| \geq x) \geq Cx^{-\alpha} \) for \( x \geq 1 \). (See Exercise 7.5 in Chapter 2 of Durrett.) Note that this is false for \( \alpha = 2 \). Assume that with probability 1, the paths \( X(t, \omega) \) are right continuous and have left limits. (Such paths are called cadlag, which is an acronym from the French phrase describing these properties.)

(a) Show that for each fixed \( t \), \( X(s) \) is a.s. continuous at \( s = t \), i.e., \( X \) has no fixed times of discontinuity.
(b) Express
\[
P \left( \max_{1 \leq k \leq n} \left| X \left( \frac{k}{n} \right) - X \left( \frac{k-1}{n} \right) \right| \geq \epsilon \right)
\]
in terms of the distribution of \( X(1) \).
(c) Using (b), prove that for \( \alpha \in (0, 2) \),
\[
P(X(t) \text{ is continuous on } [0, 1]) = 0.
\]

III. Let \( B(t) \) be standard Brownian motion, and take \( f \in L_1(R) \) with \( \int f(x)dx = 1 \). Use the method of moments to show that
\[
\frac{1}{\sqrt{t}} \int_0^t f(B(s))ds \to |Z|
\]
in distribution, where \( Z \) is \( N(0, 1) \). Note that if \( f = 1_A \) for some set \( A \) of Lebesgue measure 1, this gives the limiting distribution of the occupation time of \( A \) up to time \( t \).