Problem 1. Suppose $\phi : R^n \to R^n$ is continuously differentiable in a neighborhood of the origin and satisfies $\phi(0) = 0, |\phi'(0)| < 1$. Show that there is an $\epsilon > 0$ so that

$$X = \{x : |x| < \epsilon\}$$

satisfies the hypotheses of Theorem 9.23.

Problem 2. Suppose that $K(x, y)$ is a function defined for $x, y \in [0, 1]$. Find reasonable conditions on K so that the contraction principle can be used to show that for every $g \in C[0, 1]$ there exists a unique $f \in C[0, 1]$ that satisfies the Fredholm integral equation

$$f(x) = \int_0^1 K(x, y) f(y) dy + g(x), \quad x \in [0, 1].$$

Prove that this is true under your conditions.