(15) 1. (a) Define: “$x$ is a limit point of $E$”.

Let $E'$ be the set of limit points of $E$. Decide whether each of the following statements is true or false. If true, prove it; if false, give a counterexample.

(b) $(E \cap F)' \subset E' \cap F'$

c) $(E \cap F)' \supset E' \cap F'$
2. Let $\mathcal{N} = \{1, 2, \ldots\}$ be the set of natural numbers, $\mathcal{F}$ be the collection of finite subsets of $\mathcal{N}$, and $\mathcal{S}$ be the collection of all subsets of $\mathcal{N}$.

(a) Prove that $\mathcal{F}$ is countable.

(b) Find a one-to-one correspondence between $\{0, 1\}^\mathcal{N}$ and $\mathcal{S}$.

(c) Is $\mathcal{S}$ countable or uncountable? Why?

3. Decide whether each of the following statements is true or false. No explanation is needed.

(a) $\mathbb{Q}$, the set of rationals, has the least upper bound property. Answer:

(b) The Cantor set contains both rational and irrational points. Answer:

(c) Every metric space has at most finitely many subsets that are both open and closed. Answer:

(d) Every subset of a general metric space that is closed and bounded is compact. Answer:

(e) The union of two compact sets is compact. Answer:
(15) 4. (a) Define “$K$ is compact”.

(b) Prove that every compact set is bounded.

(c) Prove directly from the definition that $[0, 1)$ is not compact in $\mathbb{R}^1$.

(20) 5. For each part, give an example of a set $A$ in $\mathbb{R}^1$ with the usual metric that has the required properties. There is no need to prove that it has those properties:
   (a) $A$ is countable and compact.

(b) $A$ and $A^c$ are both dense.

(c) $A$ is not connected, but $\overline{A}$ is.

(d) $A$ is countable and has no limit points.
(10) 6. Suppose $A \subset R^1$ is uncountable. Prove that $A$ has a limit point.

(10) 7. Show that for each $x$, $\mathcal{O} = \{y \in X : d(x, y) > 1\}$ is open.