

A NETWORK MODEL OF IMMIGRATION: ENCLAVE FORMATION VS. CULTURAL INTEGRATION

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ABSTRACT. Successfully integrating newcomers into native communities has become a key issue for policy makers, as the growing number of migrants has brought cultural diversity, new skills, but also, societal tensions to receiving countries. We develop an agent-based network model to study interacting “hosts” and “guests” and to identify the conditions under which cooperative/integrated or uncooperative/segregated societies arise. Players are assumed to seek socioeconomic prosperity through game theoretic rules that shift network links, and cultural acceptance through opinion dynamics. We find that the main predictor of integration under given initial conditions is the timescale associated with cultural adjustment relative to social link remodeling, for both guests and hosts. Fast cultural adjustment results in cooperation and the establishment of host-guest connections that are sustained over long times. Conversely, fast social link remodeling leads to the irreversible formation of isolated enclaves, as migrants and natives optimize their socioeconomic gains through in-group connections. We discuss how migrant population sizes and increasing socioeconomic rewards for host-guest interactions, through governmental incentives or by admitting migrants with highly desirable skills, may affect the overall immigrant experience.

1. Introduction. Migrating human populations have always played a significant role in history [6, 7, 12]. Individuals driven by adventurous spirits, or seeking better socio-economic opportunities, have voluntarily abandoned their original environments. Large groups of people have also been involuntarily forced from their homelands by famine, drought, religious persecution, political turmoil, human rights violations, and wars. According to the United Nations High Commissioner for Refugees (UNHCR), the number of forcibly displaced persons worldwide has been

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steadily climbing since 2011, reaching an unprecedented level of 68.5 million persons by the end of 2017. Among these, 28.5 million are asylum seekers or refugees [47]. Economic disparity enhances the pull of populations towards more developed regions; increased mobility reduces the cost in crossing national borders and geographic barriers; advanced communication technologies facilitate long distance social connections. All of these factors contribute to the massive scale of human migration observed in recent years [8].

While large-scale emigration causes brain drain and loss of labor force in “source” countries, regions receiving immigrants also face challenges in accommodating new arrivals who may follow different social, cultural, and religious norms. Mistrust between natives and migrants may arise and exacerbate over time due to inadequate infrastructure and assistance programs. A well-documented phenomenon among immigrants is that of acculturative stress [3, 5], whereby contact with another culture may lead to psychological and somatic health issues. Overall various studies of the immigrant experience describe outcomes ranging from very positive to very negative [3, 4, 30, 44]. Immigrants joining a multicultural society generally suffer from the least acculturative stress and are the best adapted, whereas those settling in less culturally tolerant communities face more challenges [4, 30]. A common observation is that those who do not adapt well, either by circumstances or lack of motivation, often become socially marginalized. Self-segregation may lead to the creation of insular communities that offer advantages to immigrants, but that also prevent them from fully integrating [3, 4, 44]. These enclaves often deepen divisions between host and immigrant groups. The attitude of the majority host population is an important predictor of how successful the adaptation process of an immigrant group will be. Hostile host communities tend to hinder adaptation, with averse majorities playing a key role in the emergence of segregated minority communities [34, 40].

The complex relationship between natives and migrants evolves over time and depends on many economic, historic, and political factors. By framing the main ingredients of this relationship in simple, quantifiable ways, mathematical models may help one to understand the implications of various mechanisms and of their synergy, and may help design intervention strategies. Agent-based mathematical models have been recently employed to study coexistence and cooperation among culturally heterogeneous populations through game theory [9, 11, 16, 26, 27, 33, 38, 41, 42, 48], opinion dynamics [10, 14, 15, 17, 18, 20, 25, 35, 50], population dynamics [1, 19, 28], and network theory [11, 16, 27, 29]. In this paper we introduce an agent-based social-network model that assumes immigrant groups have two primary objectives: to improve their socioeconomic status and to gain acceptance within their social circles. The former scenario is usually modeled by implementing game theory rules, whereby a utility function associated with socioeconomic status is to be maximized [9, 11, 26, 27, 38, 42]. The latter is typically described using opinion dynamics, whereby individuals adjust their opinions or cultural traits through social interactions [10, 14, 15, 17, 18, 20, 22, 35, 50]. Simplistic game theory models rarely yield cooperative patterns, as defectors tend to prevail if each agent is allowed to only make rational decisions for his or her own self-interest [38]. Cooperative behavior may emerge through biased decision making whereby individuals collaborate solely with those that share their same opinion. This mechanism leads to social segregation, as tight collaborations develop only within culturally homogeneous enclaves [9, 11, 16, 26, 27, 39, 41, 42]. Models of opinion dynamics on the other hand often

assume individuals seek like-minded peers, and willingly adjust to prevailing opinions [1, 22]. Minority opinions arise and persist only through ad-hoc restrictions, such as including zealots, or by imposing thresholds so that consensus is reached only if two opinions are sufficiently close [23, 39].

As a rule of thumb, game theoretic models result in uncooperative behavior; opinion dynamics leads to uniform consensus. The immigrant narrative, however, is much more nuanced with behaviors ranging from uncooperative segregation to cooperative integration, suggesting modeling should include both mechanisms. We thus introduce a network populated with interacting “guest” and “host” nodes that seek to improve their socioeconomic status while culturally adjusting to each other. Socioeconomic gains are modeled via a utility function that evolves through game theoretic rules, while the attitudes (or “opinions”) that players harbor towards others evolve through opinion dynamics. These two mechanisms are interdependent, so that attitudes towards different cultures shape utility gains, and vice versa.

We show that the main predictor of integration or segregation is given by the relationship between two timescales: that of cultural adjustment, whereby guests and hosts adapt more tolerant attitudes of each other, and that of social link remodeling, whereby players change their network connections to increase their socioeconomic rewards. In the case of slow cultural adjustment, immigrant and host communities tend to segregate as accumulation of socioeconomic wealth occurs more efficiently through insular in-group connections. Conversely, if adjustment is sufficiently fast, cross-cultural bridges may be established and sustained, allowing different cultural groups to reach “consensus” and maintain active cooperation. Another key role will be played by the fraction of immigrants joining the total population as the immigrant-to-host ratio changes the cultural adjustment timescales. As we outline below, a high immigrant ratio increases the likelihood of in-group connections and reduces communication between immigrant and host populations.

In Section 2, we introduce our network model, the mechanisms that govern the evolution of social connections, and the utility function for immigrant-host interactions. In Section 3 we examine the parameter dependence of our model and show how processes unfolding over different timescales lead to different outcomes of immigration integration. Finally, we conclude in Section 4 with a discussion on sociological and policy implications.

2. The model. Our basic model consists of a network whose nodes symbolize immigrant or native agents connected by edges that represent social links. Each node is also associated with an attitude and a utility function that depend on its connections and that determine an agent’s socioeconomic status. Over time, nodes change their connections and attitudes as they seek to increase their utility; as a result the network evolves towards integration or segregation between immigrant and host communities.

2.1. Network. Within our network model a node represents a social unit, such as an individual or a collection of individuals, and is labeled as a “guest” or a “host”, depending on whether it belongs to the immigrant or native group. Each node, indexed by i , is characterized by an “attitude” variable x_i^t at time t , which varies between $-1 \leq x_i^t \leq 0$ for guest nodes and in $0 \leq x_i^t \leq 1$ for host nodes. Hence the sign of x_i^t is used to distinguish the group identity of the node. The magnitude $|x_i^t|$ indicates the degree of hostility that node i harbors towards those belonging to the other group. Thus, $x_i^t \rightarrow 0^\pm$ characterizes most receptive guests

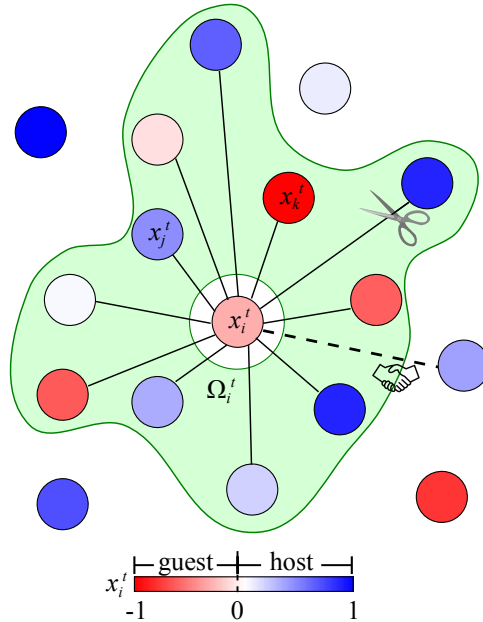


FIGURE 1. Model diagram. Each node i is characterized by a variable attitude $-1 \leq x_i^t \leq 1$ at time t . Negative values, depicted in red, indicate guest nodes; positive values represent hosts, colored in blue. The magnitude $|x_i^t|$ represents the degree of hostility of node i towards members of the other group. Each node is shaded accordingly. All nodes j, k linked to the central node i represent the green-shaded social circle Ω_i^t of node i at time t . The utility U_i^t of node i depends on its attitude relative to that of its m_i^t connections in Ω_i^t and on m_i^t . Nodes maximize their utility by adjusting their attitudes x_i^t and by establishing or severing connections, reshaping the network over time.

or most hospitable hosts, while $x_i^t = \pm 1$ represents the highest level of xenophobia. Moreover, we define Ω_i^t as the “social circle” of node i at time t , which is a set containing all nodes directly connected to node i at time t . We assume that there are a fixed number of N_h host and N_g guest nodes, with varying attitudes. All nodes $N = N_h + N_g$ seek to maximize their utility function as defined below.

2.2. Utility function. The dynamics of our network is driven by the utility function U_i^t assigned to each node i . Each player seeks to maximize U_i^t by shifting its attitude x_i^t , and by forging and severing connections with other nodes. We model the utility U_i^t of node i at time t via two components: a reward function u_{ij}^t for interacting with node j , and a cost function $c(m_i^t)$ for maintaining m_i^t connections so that

$$\begin{aligned}
 U_i^t &= \sum_{j \in \Omega_i^t} u_{ij}^t - c(m_i^t) \\
 &= \sum_{j \in \Omega_i^t} A_{ij} \exp\left(-\frac{(x_i^t - x_j^t)^2}{2\sigma}\right) - \exp\left(\frac{m_i^t}{\alpha}\right).
 \end{aligned} \tag{1}$$

The pairwise reward function u_{ij}^t depends on the attitude difference $|x_i^t - x_j^t|$ between connected nodes i and j ; the smaller the attitude difference, the higher the reward. For a pair of nodes from the same group, *i.e.* if both i and j are hosts or immigrants, maximizing u_{ij}^t implies $x_i^t = x_j^t$ leading to consensus within the group. If i and j are nodes from different groups, u_{ij}^t is maximized by both sides adopting more cooperative attitudes such that $x_i^t \rightarrow 0^-$ and $x_j^t \rightarrow 0^+$. Hence, the value of x_i^t that will maximize U_i^t will depend on the composition of Ω_i^t and the attitudes x_j^t of its members. The parameter σ controls the sensitivity of the reward, while the amplitude A_{ij} specifies the maximum reward attainable when $x_i^t = x_j^t$. In principle, A_{ij} may depend on the specific socioeconomic attributes of the interacting i, j pair. For simplicity we let A_{ij} be one of two discrete levels; $A_{ij} = A_{\text{in}}$ for in-group interactions, where nodes i and j belong to the same group, both hosts or both migrants, and $A_{ij} = A_{\text{out}}$ for out-group interactions between nodes i and j of different groups. The cost function c in Eq. 1 is a function of $m_i^t = |\Omega_i^t|$, the number of connections sustained by node i at time t , which by definition is also the cardinality of the social-circle set Ω_i^t . We assume that the cost to maintain connections increases exponentially with m_i^t through a scaling coefficient α . A smaller α value results in a steeper increase of cost, leading to fewer average connections per node. Note that such a cost function penalizes nodes with too many connections, suppressing the likelihood of “hub” nodes of high connectivity, a hallmark of small world networks that characterizes many real world social networks. In more realistic settings, the cost of maintaining social connections depends on more nuanced characteristics of each individual (wealth, fame, age, community status), allowing some to sustain higher degrees of connectivity than others. For simplicity our model does not include these considerations.

2.3. Mechanisms of model evolution. At each time step, each node i seeks to increase its utility U_i^t by adding or cutting connections and adjusting its attitude x_i^t . We model this process as a series of stochastic events through the following steps:

1. At time t , randomly pick the “active” node i to make a decision.
2. Randomly pick another node $j \neq i$.
 - If i and j are connected, *i.e.*, $j \in \Omega_i^t$, check whether breaking the i - j connection increases U_i^t for node i . If it does, break the i - j connection.
 - If $j \notin \Omega_i^t$, check whether adding an i - j connection increases U_i^t for node i . If it does, add the i - j connection.
3. Randomly pick a connected node $\ell \in \Omega_i^t$ via a reward-weighted probability

$$p_\ell = \frac{u_{i\ell}^t}{\sum_{\ell \in \Omega_i^t} u_{i\ell}^t}. \quad (2)$$

4. Determine x_i^{t+1} using x_i^t, x_ℓ^t

$$x_i^{t+1} = \begin{cases} \min \left(0, x_i^t + \frac{x_\ell^t - x_i^t}{\kappa} \right) & \text{for } x_i^t < 0 \text{ (guest),} \\ \max \left(0, x_i^t + \frac{x_\ell^t - x_i^t}{\kappa} \right) & \text{for } x_i^t > 0 \text{ (host),} \end{cases} \quad (3)$$

Symbol	Description	default values
x_i	attitude	-1 to 1
A_{in}	maximal utility through in-group connection	10
A_{out}	maximal utility through out-group connection	1 to 100
σ	sensitivity to attitude difference	1
κ	attitude adjustment timescale	100 to 1000
α	cost of adding connections	3
N	total population	2000
N_{g}	guest population	20 to 200
N_{h}	host population	$N - N_{\text{g}}$

TABLE 1. List of variables and parameters of the model.

where κ is the timescale associated with attitude adjustment. Large values of κ indicate longer adaptation times. We select different nodes $j \neq \ell$ for remodeling network connections and adjusting attitudes to avoid the emergence of any systematic biases.

5. Advance time $t \rightarrow t + (1/N)$ and repeat steps 1–4.

In the above steps, all unweighted random selections are made through a uniform probability. As presented, our algorithm alternates between remodeling network connections and making attitude adjustments. Note that when steps 1–4 are repeated on all N nodes, t advances to $t + 1$, and that, on average, each node makes decisions once within this unitary time step. Thus, the timescale for network remodeling is one. The timescale for attitude adjustment, instead, is given by κ scaled by the probability for node i to be paired with node ℓ carrying a different attitude. We can approximate this probability as the fraction of out-group connections, N_{g}/N for hosts and N_{h}/N for guests, so that the guest adjustment timescale τ_{h} can be estimated by $\tau_{\text{h}} \sim \kappa N/N_{\text{g}}$, and the host adjustment timescale τ_{g} by $\tau_{\text{g}} \sim \kappa N/N_{\text{h}}$.

An important observation is that U_i^t can reach its maximum U_i^{max} if $|x_i^t - x_j^t| \rightarrow 0$ within connected components of the network. This can be achieved in two different ways: i) through actual consensus where all nodes carry a neutral attitude $x_i^t \rightarrow 0$ so that in-group and out-group connectivities are equally likely, or ii) through a segregated network with homogeneous clusters made of all guests or all hosts, where non-zero but uniform attitudes are maintained in each cluster, so that $|x_i - x_j^t| \rightarrow 0$ does not necessarily imply $x_i^t \rightarrow 0$. Although these two different network configurations lead to the same maximal utility, only the first one will be considered a true hallmark of harmonious integration, since attitudes are the most open on both sides, and there is minimal differentiation between intra-group or out-group connectivity. The second case instead represents the creation of parallel societies, with each group self-segregating into its own homogeneous enclave, maintaining little contact with “the other”.

2.4. Initial conditions. All model parameters and typical values are listed in Table 1. Unless otherwise specified, our network simulations are performed using the initial conditions described here. We mostly simulate $N = 2000$ nodes, within which $N_{\text{h}} = 1800$ are hosts and $N_{\text{g}} = 200$ are guests. In Section 3.1 we also simulate the setting of $N_{\text{g}} = 20$ and $N_{\text{h}} = 1980$ to examine the effect of extremely small fractions of guests. The initial attitudes are set at $x_i^0 = 1$ for all host nodes, and $x_i^0 = -1$ for all guest nodes, assuming that before the two groups make any contact they have minimal knowledge on how to coexist. For initial connections, we mostly use the

following two extreme and opposite scenarios. One is that host and guest nodes are randomly connected with uniform probability, yielding on average ten connections per node at $t = 0$. The other is that hosts are connected to each other and that no guests are present. Host connectivity is determined by allowing the system to equilibrate in the absence of guests, representing the natural state of the community before the arrival of immigrants. Guests are introduced at $t = 0$ as nodes without any links to either hosts or fellow guests. Note that because of the definition of the utility function in Eq. 1, and because we allow the host community to equilibrate prior to inserting guests, we expect each host to be connected to an average number of $\alpha \ln(\alpha A_{\text{in}})$ other hosts at $t = 0$. The first initial condition scenario represents a perfectly executed welcoming program for immigrants, providing with them sufficient social ties to connect to the native community. In the second initial condition scenario, such a welcoming program does not exist at all, and guests arrive in a completely foreign environment.

3. Results. Figure 2 shows two representative outcomes of our network model at steady state. In Fig. 2a guests (red circles) and hosts (blue circles) segregate and maintain highly hostile attitudes, as illustrated by the dark red and blue shades of the right hand panel. Cross-group utilities at the beginning of simulations yield low rewards which do not increase over time, leading to the severing of all ties between hosts and guests at $t \rightarrow \infty$. In Fig. 2b guests adopt more cooperative attitudes as represented by the lighter red colors. Such attitudes increase cross-group rewards so that guests and hosts stay mixed. Hosts will also become more cooperative, although at slower timescales than guests.

The two configurations shown in Fig. 2 represent two ways through which U_i^t in Eq. 1 is maximized. The configuration in Fig. 2a arises by cutting all cross-group links to form enclaves, within which guests and hosts adopt uniform but different attitudes $x_{i,\text{guest}} \neq x_{i,\text{host}} \neq 0$. The configuration in Fig. 2b emerges through cooperative attitudes $x_{i,\text{guest}} = x_{i,\text{host}} = 0$ for all players. Both lead to $|x_i^t - x_j^t| \rightarrow 0$ as $t \rightarrow \infty$. To which of these two basins of attraction society converges, will depend on parameter choices and initial conditions as discussed below.

3.1. Maximizing utilities via network remodeling and attitude adjustment.

For a more quantitative perspective, we now examine how the utility function U_i^t and the attitude profiles x_i^t vary over time in some sample simulations. We set the model parameters to $\alpha = 3$, $A_{\text{in}} = A_{\text{out}} = A = 10$, and $\sigma = 1$, and let κ vary between 100 and 1000 with $N = 2000$ and $N_{\text{g}} = 200$ or $N_{\text{g}} = 20$. The assumption $A_{\text{in}} = A_{\text{out}} = A$ leads to a maximum in the utility $U_i^t = U_i^{\text{max}} = \alpha A [\ln(\alpha A) - 1]$ which is reached if all connected nodes conform their attitude so that $|x_i^t - x_j^t| \rightarrow 0$ for any linked i, j pair and when each node i has $m_i^t = m_{\text{opt}} = \alpha \ln(A_{\text{in}} \alpha)$ links. For our chosen parameters, $m_{\text{opt}} = 10$ connections and $U_i^{\text{max}} = 72$.

Since we are interested in how immigrants adapt to their host environment, we will mainly focus on quantities associated with guest nodes. Although host node properties will also dynamically evolve, relative changes to their attitudes x_i^t and connections will be much slower than that of guests due to their overwhelming majority. Initial conditions are chosen so that guests and hosts are randomly connected to each other as described in Section 2.4. For $N_{\text{g}} = 200$ the relatively large number of guests allows for segregated clusters to emerge and persist with $m_{\text{opt}} = 10$ in-group connections. For $N_{\text{g}} = 20$ the low number of guests either leads to smaller in-group guest clusters with less-than-optimal number of connections ($m_{\text{opt}} < 10$),

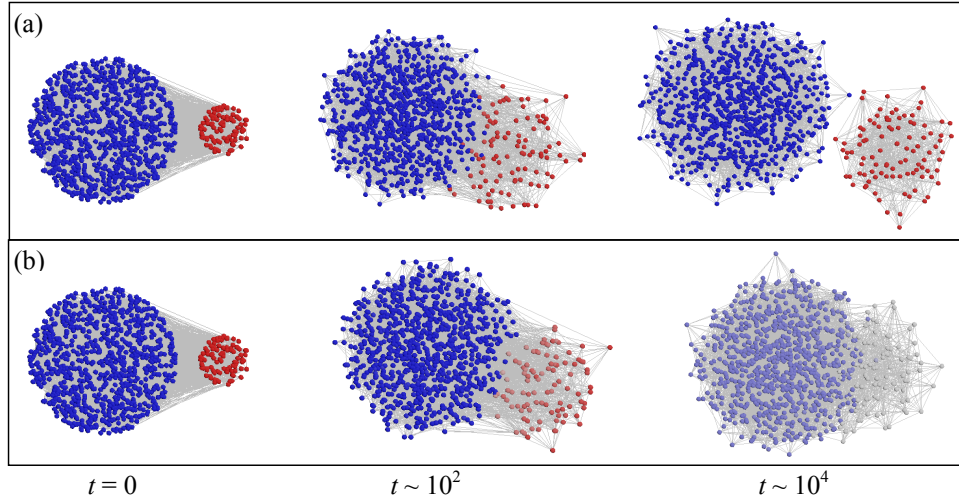


FIGURE 2. Simulated network dynamics leading to (a) complete segregation, and (b) integration between guest (red) and host (blue) populations. Shading of node colors represents the degree of hostility $|x_i^t|$ of node i towards those of its opposite group, according to the color scheme shown in Fig. 1. Initial conditions are randomly connected guest and host nodes with attitudes $x_{i,\text{guest}}^0 = -1$ and $x_{i,\text{host}}^0 = 1$. Other parameters are $N_h = 900, N_g = 100, \alpha = 3, A_{\text{in}} = A_{\text{out}} = 10, \sigma = 1$. The two panels differ only for κ , the attitude adjustment timescale, with $\kappa = 1000$ in panel (a) and $\kappa = 100$ in panel (b). (a) For slowly changing attitudes ($\kappa = 1000$), hostile attitudes persist over time, eventually leading to segregated clusters. (b) For fast changing attitudes ($\kappa = 100$), guests initially become more cooperative, as shown by the lighter red colors. Over time, a more connected host-guest cluster arises with hosts eventually adopting more cooperative attitudes as well.

or forces host-guest mixture to reach $m_{\text{opt}} = 10$. We will first examine network remodeling and attitude adjustment independently of each other, and later the interplay between the two mechanisms.

Figure 3 shows the temporal evolution of the average utility $\langle U_i^t \rangle_{\text{guest}}$ per guest node and the average attitude of guests and hosts $\langle x_i^t \rangle_{\text{guest}}, \langle x_i^t \rangle_{\text{host}}$ for sample simulations of $N_g = 200$ (Figs. 3a and 3b) and $N_g = 20$ (Figs. 3c and 3d) guest nodes with $N = 2000$ total nodes. In the red-solid curves we only allow for network remodeling and deactivate attitude adjustment. Vice-versa, in the blue-dashed ($\kappa = 100$) and green-dotted curves ($\kappa = 1000$) we only allow for attitude adjustment and deactivate network remodeling. Finally, the purple dotted-dashed curve ($\kappa = 100$) and the magenta-double-dotted-dashed curve ($\kappa = 1000$) are results from the full model, where both network remodeling and attitude adjustment are implemented.

As can be seen in Fig. 3a for $N_g = 200$, $\langle U_i^t \rangle_{\text{guest}}$ increases over time towards $U_i^{\text{max}} = 72$ for all five chosen cases. When only network remodeling is allowed (red-solid curve), $\langle U_i^t \rangle_{\text{guest}}$ increases quickly at the onset of the dynamics as nodes efficiently exchange low-utility, out-group connections for high-utility, in-group ones.

As the number of exchanges nears completion, $\langle U_i^t \rangle_{\text{guest}}$ increases at a slower rate, until it converges to the steady state at $U_i^{\text{max}} = 72$ with optimal, high-utility connections that are mostly in-group. Guests have established their own self segregated communities and thrive within it. When only attitude adjustment is activated (blue-dashed $\kappa = 100$ and green-dotted $\kappa = 1000$ curves), nodes can only change their attitude and not their connections, hence they tend to evolve towards conformity ($|x_i^t - x_j^t| \rightarrow 0$ for all nodes i, j). Note that if i, j are a guest-host pair respectively, conformity will only arise from $x_{i,\text{host}} \rightarrow 0^-, x_{j,\text{host}} \rightarrow 0^+$. Since $\langle U_i^t \rangle_{\text{guest}}$ depends solely on attitude adjustment, its dynamics will vary on the same timescale as x_i^t , given by $\tau_g = N/N_h\kappa$. In the case of fast attitude adjustment (blue-dashed curve for $\kappa = 100$), the early rise of $\langle U_i^t \rangle_{\text{guest}}$ can be more pronounced than in the case of network remodeling (red-solid curve), as can be seen for short times ($t \lesssim 2000$) in Fig. 3a. However, the utility at steady state $\langle U_i^{\text{ss}} \rangle_{\text{guest}}$ under attitude adjustment is lower than under network remodeling, regardless of κ . This is because when only attitude adjustment is allowed, network connections cannot be rearranged, resulting in a less-than-optimal connectivity that changes to x_i^t can only partially alleviate. Having network adjustment as the sole mechanism at play allows for more flexibility, since, although x_i^t cannot change, a given node can actively search for others with similar attitude and even increase its number of connections. We verified that when only one of the two mechanisms is allowed, attitude adjustment consistently leads to less optimal outcomes compared to network remodeling for a number of parameter choices and initial conditions.

These trends are confirmed and better elucidated by inspecting the average attitudes of guests $-1 \leq \langle x_i^t \rangle_{\text{guest}} \leq 0$ and hosts $0 \leq \langle x_i^t \rangle_{\text{host}} \leq 1$ as a function of time in Fig. 3b. We use the same parameter sets and initial conditions as in Fig. 3a and the same color-coding scheme. The red-solid curves correspond to the case where we only allow for network readjustment and attitudes stay unmodified, so $\langle x_i^t \rangle_{\text{guest}} = -1$ and $\langle x_i^t \rangle_{\text{host}} = 1$ for all times. The blue-dashed and green-dotted curves, where only attitude adjustment is allowed show that as t increases, $\langle x_i^t \rangle_{\text{guest}} \rightarrow 0^-$ at a faster rate, and that $\langle x_i^t \rangle_{\text{host}} \rightarrow 0^+$ at a much slower one. This is easily understood. Since nodes are not allowed to rewire their connections, they can only adapt their attitudes as discussed above, and provided the network is connected and no isolated clusters exist, all nodes will eventually conform to $x_i^t \rightarrow 0$. However, being a numerical minority in the network, guests, for which $x_{i,\text{guest}}^t \leq 0$, will share a large number of connections with hosts, for which $x_{i,\text{host}}^t \geq 0$. Under this condition, the adaptation rules presented in Sec. 2.3 drive guests towards conformity more than hosts, so that $\langle x_i^t \rangle_{\text{guest}} \rightarrow 0^-$ faster than $\langle x_i^t \rangle_{\text{host}} \rightarrow 0^+$. Hence, the early increases in $\langle U_i^t \rangle_{\text{guest}}$ when only attitude adjustment is allowed and observed in Fig. 3a (blue-dashed $\kappa = 100$, and green-dotted $\kappa = 1000$ curves) can be attributed to fast adaptation of guests with time scale $\tau_g = \kappa N/N_h$, and the later increases to slow adaptation of hosts with time scale $\tau_h = \kappa N/N_g \gg \tau_g$.

The dynamics of the full model (purple-dotted-dashed $\kappa = 100$, and magenta-double-dotted-dashed $\kappa = 1000$ curves) depend on the interplay between the two mechanisms at play, attitude adjustment and network remodeling, and the respective timescales in gaining utility. From Fig. 3a, $\langle U_i^t \rangle_{\text{guest}}$ for the full model with fast attitude adjustment (purple-dotted-dashed, $\kappa = 100$) follows the attitude adjustment (blue-dashed, $\kappa = 100$) curve at early times, later shifting towards the network remodeling (red-solid) curve. Guests thus find it more advantageous to first adjust their attitudes, and then modify their network connectivity. Similarly, Fig. 3b shows

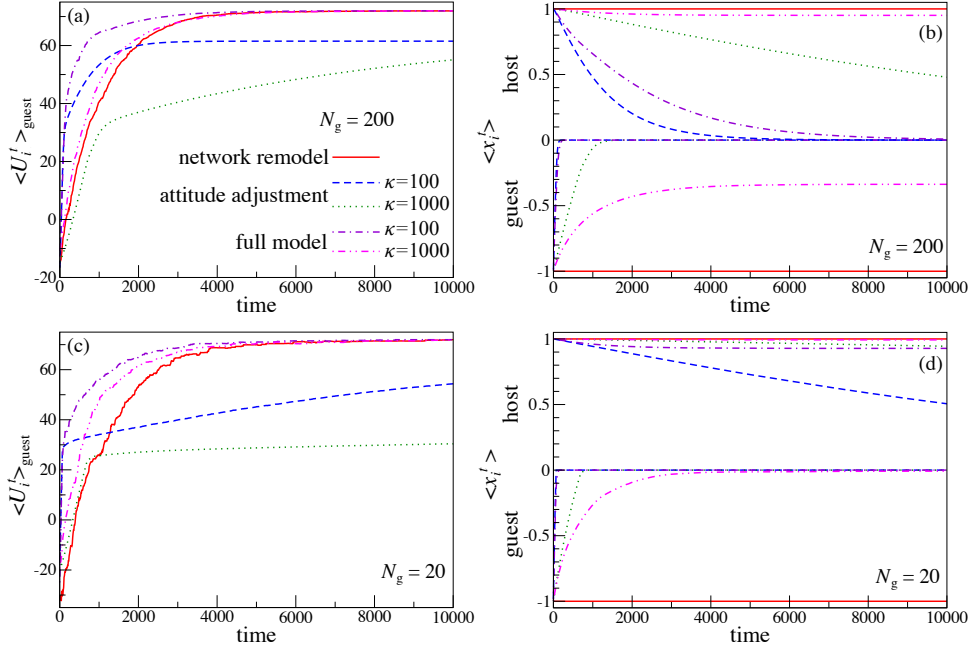


FIGURE 3. Dynamics of the average utility per node $\langle U_i^t \rangle_{\text{guest}}$ in panels (a) and (c), and of the average attitudes $\langle x_i^t \rangle_{\text{guest}}, \langle x_i^t \rangle_{\text{host}}$ in panels (b) and (d) for $N_g = 200$ (a,b) and $N_g = 20$ (c,d) guests in a total population of $N = 2000$ nodes. Parameters are $\alpha = 3$, $A_{\text{in}} = A_{\text{out}} = 10$, and $\sigma = 1$, and $\kappa = 100$ (faster) and $\kappa = 1000$ (slower) attitude adjustment. Initial attitudes are $x_{i,\text{host}}^0 = 1$ and $x_{i,\text{guest}}^0 = -1$, with random connections between nodes so that on average each node is connected to $m_i^0 = 10$ others at $t = 0$, representing full insertion of guests into the community. Network remodeling (solid-red curve) and attitude adjustment (blue-dashed and green-dotted curves) are considered separately; their interplay is illustrated in full model simulations (purple-dot-dashed and magenta-double-dotted-dashed). Utility is increased in all cases, but attitude adjustment is more efficient at the onset due to the initially set cross-group connections. Network remodeling allows for higher utilities at longer times. For the full model, fast adjustment ($\kappa = 100$) leads to well integrated societies for $N_g = 200$ as $t \rightarrow \infty$, given that $\langle x_i^t \rangle_{\text{host}} \rightarrow 0^+$ and $\langle x_i^t \rangle_{\text{guest}} \rightarrow 0^-$; for $N_g = 20$ hosts and guests segregate, with guests adopting collaborative attitudes, $\langle x_i^t \rangle_{\text{host}} \rightarrow 0.93$ and $\langle x_i^t \rangle_{\text{guest}} \rightarrow 0^-$. Under slow adjustment ($\kappa = 1000$) hosts and guests will remain hostile and segregated with $\langle x_i^t \rangle_{\text{host}} \rightarrow 0.95$, $\langle x_i^t \rangle_{\text{guest}} \rightarrow -0.34$ for $N_g = 200$ and $\langle x_i^t \rangle_{\text{host}} \rightarrow 0.99$, $\langle x_i^t \rangle_{\text{guest}} \rightarrow 0^-$ for $N_g = 20$.

$\langle x_i^t \rangle_{\text{guest}} \rightarrow 0^-$ as $t \rightarrow \infty$, following the curve where only attitude adjustment is allowed. The convergence of $\langle x_i^t \rangle_{\text{host}} \rightarrow 0^+$ is slower because network remodeling allows the many hosts to replace their relatively few out-group connections with

conspecifics. Eventually however, both guests and hosts converge towards integration, with $\langle x_i^t \rangle_{\text{guest}} \rightarrow 0^-$, $\langle x_i^t \rangle_{\text{host}} \rightarrow 0^+$. In contrast, $\langle U_i^t \rangle_{\text{guest}}$ for the full model with slow attitude adjustment (magenta-double-dotted-dashed $\kappa = 1000$) follows the network remodeling (red-solid) curve at all times. Here, guests find it more advantageous to change their connectivity, preferentially creating links to other guest nodes, rather than modify their attitudes towards host communities. Indeed attitudes converge to $\langle x_i^t \rangle_{\text{guest}} \rightarrow -0.34$ and $\langle x_i^t \rangle_{\text{host}} \rightarrow 0.95$ as $t \rightarrow \infty$, with no further attitude adjustment possible.

This example illustrates the central role played by κ in the dynamics: low values of κ , indicating relatively short times for attitude adjustment τ_g, τ_h , lead to harmonious societies with $x_i \rightarrow 0$ for all nodes, while larger values of κ , indicating longer times for attitude adjustment, lead to segregated communities.

In Figs. 3c and 3d we show $\langle U_i^t \rangle_{\text{guest}}$ and $\langle x_i^t \rangle_{\text{guest}}$ for a smaller immigrant population, $N_g = 20$ and the same parameters as in Figs. 3a and 3b. We observe the same qualitative increase of utility in each of the five cases as discussed above. Discrepancies with plots obtained for $N_g = 200$ mainly emerge when only attitude adjustment is allowed (blue-dashed $\kappa = 100$, and green-dotted $\kappa = 1000$ curves). Here, the early increase of utility is faster than for $N_g = 200$, but steady state is reached at a much slower rate. The overwhelming majority of hosts drives guests to rapidly adjust their attitudes, increasing $\langle U_i^t \rangle_{\text{guest}}$ at short times. By the same token, the host majority will not significantly change its attitude, so that guests can further increase their utility only by remodeling their connectivity. Indeed, the corresponding curves in Fig. 3d show guests rapidly converging to $\langle x_i^t \rangle_{\text{guest}} \rightarrow 0^-$ for all cases, while $\langle x_i^t \rangle_{\text{host}}$ does not. Note that as long as the network is initially connected and no isolated clusters exist, when only attitude adjustment is allowed, $\langle x_i^t \rangle_{\text{host}} \rightarrow 0^+$ as $t \rightarrow \infty$, although the process may be slow. For $N_g = 20$, due to the low number of guests, there is a higher probability than for $N_g = 200$ of initiating the model with isolated host-only clusters. For these clusters, if only attitude adjustment is allowed, attitudes will stay quenched at $\langle x_i^t \rangle_{\text{host}} \rightarrow 1$. As a result, the overall $\langle x_i^t \rangle_{\text{host}}$ will converge towards a non zero value.

In the case of the full model (purple-dotted-dashed, $\kappa = 100$ and magenta-double-dotted-dashed $\kappa = 1000$) we see a similar trend $\langle x_i^t \rangle_{\text{guest}} \rightarrow 0^-$, while $\langle x_i^t \rangle_{\text{host}} \rightarrow 0.93$ for $\kappa = 100$, and $\langle x_i^t \rangle_{\text{host}} \rightarrow 0.99$ for $\kappa = 1000$ as $t \rightarrow \infty$. Segregated host communities arise, with the numerically lesser guests adapting to the majority.

3.2. Quantifying outcomes of integration. The above results lead us to seek measures to better understand the topology of the network as a function of time, specifically from the guest standpoint. To this end, we introduce an integration index I_{int}^t as the relative number of out-group connections of a guest node, averaged over all nodes, and scaled by the host population fraction

$$I_{\text{int}}^t \equiv \frac{N}{N_h} \left\langle \frac{m_{i,\text{out}}^t}{m_i^t} \right\rangle_{\text{guest}}. \quad (4)$$

Here, N_h/N is the time independent host population fraction and $m_{i,\text{out}}^t$ is the number of out-group connections; the ratio $m_{i,\text{out}}^t/m_i^t$ is averaged over all guest nodes. A guest-only enclave for which $m_{i,\text{out}}^t = 0$ leads to $I_{\text{int}} = 0$. Conversely, in a uniformly mixed guest-host configuration, $m_{i,\text{out}}^t/m_i^t$, should not be too dissimilar from the host population fraction N_h/N , leading to $I_{\text{int}}^t \rightarrow 1$. As defined, $0 \leq I_{\text{int}}^t \leq$

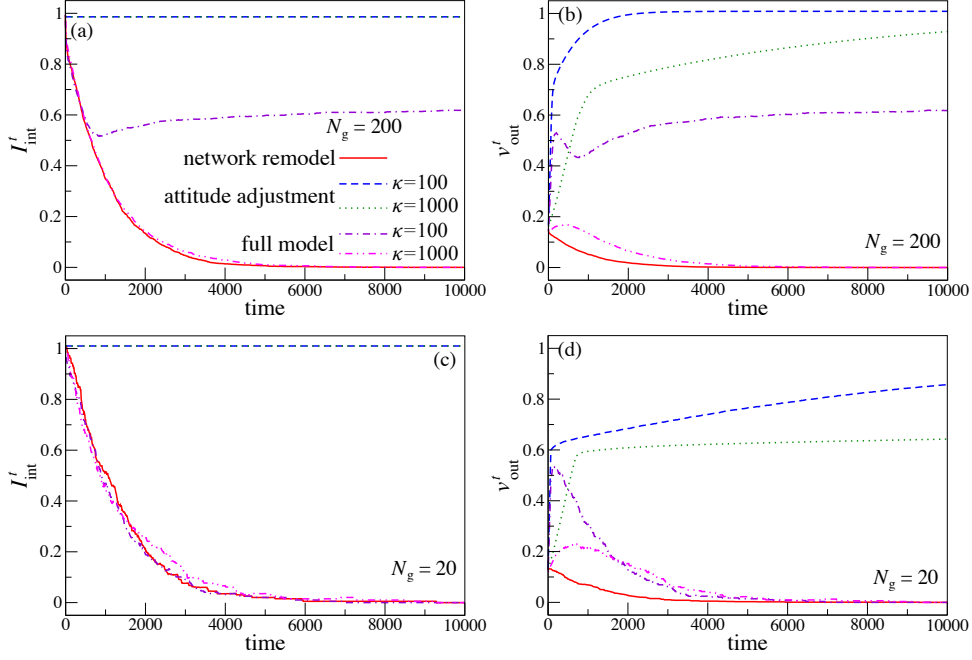


FIGURE 4. Dynamics of the integration index I_{int}^t in panels (a) and (c) and of the out-group reward fraction v_{out}^t in panels (b) and (d). Parameters and initial conditions are the same as in Fig. 3. (a, b) Large migrant population $N_g = 200$. Here, $I_{\text{int}}^t \rightarrow 0$ and $v_{\text{out}}^t \rightarrow 0$ at long times when only network remodeling is allowed, and nodes seek links with conspecifics. If only attitude adjustment is allowed, I_{int}^t remains fixed due to the quenched network connectivity, while v_{out}^t increases as guests and hosts adopt more cooperative attitudes. For the full model, slow attitude changes ($\kappa = 1000$) lead to segregation and $I_{\text{int}}^t \rightarrow 0$, $v_{\text{out}}^t \rightarrow 0$ as $t \rightarrow \infty$. Fast attitude changes ($\kappa = 100$) lead to non-zero values of I_{int}^t and v_{out}^t , indicating a more cooperative society. (c, d) Small migrant population $N_g = 20$. Results are similar to the previous case except for the full model where $I_{\text{int}}^t \rightarrow 0$, $v_{\text{out}}^t \rightarrow 0$ as $t \rightarrow \infty$ for both $\kappa = 1000$ and $\kappa = 100$. For low values of N_g segregation arises under both fast and slow attitude changes.

N/N_h . At $I_{\text{int}}^{\text{max}} = N/N_h \geq 1$ guest nodes preferentially connect to hosts, shunning other guest nodes. We refer to this outcome as reverse segregation.

While I_{int}^t measures the connectivity between guest and host nodes, another relevant measure is the fraction of the reward u_{ij}^t that arises from cross-group interactions. This is important, as guests connecting predominantly to host nodes may not necessarily be an indicator of balanced socioeconomic growth. For example, even for large values of $I_{\text{int}}^t \gtrsim 1$ hosts may share large rewards among themselves but very little with guests, representing a two-track society where guests, although connected, are not part of the mainstream socioeconomic activity.

In a perfect scenario, guests and hosts form an all-connected network, with $N_g N_h$ out-group, host-guest connections among the total $N(N-1)/2$ edges. If the reward is distributed equally among all edges, the ratio of out-group connections is given by $2N_g N_h / N(N-1)$. We thus define an out-group reward fraction v_{out}^t as follows

$$v_{\text{out}}^t \equiv \frac{\sum_{\substack{i \in \text{guests} \\ j \in \text{hosts}}} u_{ij}^t}{\sum_{\substack{i \in \text{all nodes} \\ (j \neq i) \in \text{all nodes}}} u_{ij}^t / 2} \cdot \frac{N(N-1)}{2N_g N_h}. \quad (5)$$

The first term on the right-hand side is the fraction of reward shared between guests and hosts with respect to the total. We then renormalize this quantity by the ratio $2N_g N_h / N(N-1)$ derived above for the perfectly mixed scenario. As a result, $v_{\text{out}}^t = 1$ indicates a connected network with no isolated clusters and with rewards equally spread among all nodes. Instead, $v_{\text{out}}^t = 0$ points to complete segregation, where no socioeconomic reward comes from cross-group activities. Note that v_{out}^t can exceed unity if the cross-group economy is more flourishing than intra-group growth.

The dynamics of I_{int}^t and v_{out}^t under the same parameter choices and mechanisms used to plot Fig. 3 are shown in Fig. 4. We first discuss the case of $N_g = 200$, in Figs. 4a and 4b. If we allow only for network remodeling (red-solid curves), the system will evolve towards segregation ($I_{\text{int}}^t \rightarrow 0$ in Fig. 4a and $v_{\text{out}}^t \rightarrow 0$ in Fig. 4b). Here, since attitudes cannot change, nodes will maximize their utility through in-group connections and by creating insular communities. In the blue-dashed and green-dotted curves we deactivate network remodeling and only allow for attitude adjustment, with $\kappa = 100, 1000$ respectively. As can be seen from Fig. 4a $I_{\text{int}}^t \simeq 1$ at all times since the random connections assigned at $t = 0$ are fixed and guest and host nodes remain well mixed in time. Fig. 4b shows that as cooperative attitudes emerge, cross-group rewards v_{out}^t increase. In the case of fast attitude adjustment $\kappa = 100$ (blue-dashed curve), when nodes are completely cooperative, $v_{\text{out}}^t \rightarrow 1$ as $t \rightarrow \infty$, while in the case of slow attitude adjustment $\kappa = 1000$ (green-dotted curve) convergence to $v_{\text{out}}^t \rightarrow 1$ is slower.

Results for the full model reveal the subtle interplay between network remodeling and attitude adjustment. At early times I_{int}^t follows the network remodeling case only (red-solid curve) for both $\kappa = 100$ and $\kappa = 1000$. In both scenarios guests progressively sever their ties to hosts, due to their low utility. At the same time, attitude adjustment increases cooperativity on the given initial connections and v_{out}^t temporarily increases. Eventually ineffective cross-group connections are completely eliminated under slow attitude adjustment (magenta-double-dotted-dashed, $\kappa = 1000$) where $I_{\text{int}}^t \rightarrow 0$ and $v_{\text{out}}^t \rightarrow 0$ as $t \rightarrow \infty$. Under fast attitude adjustment (purple-dot dashed, $\kappa = 100$) instead cross-group connections contribute to the utility, so that $I_{\text{int}}^t \rightarrow 0.6$ and $v_{\text{out}}^t \rightarrow 0.6$ as $t \rightarrow \infty$. Note that I_{int}^t and v_{out}^t converge to the same value as $t \rightarrow \infty$ since $|x_i^t - x_j^t| \rightarrow 0$ for both in-group and out-group connections. As a result, the distribution of rewards directly reflects the fraction of cross-group connections.

Taken together with results shown in Fig. 3a and 3b, the above dynamics confirm the crucial role played by κ , the attitude adjustment timescale, in determining societal outcomes. For the chosen parameters and when the full model is considered, more rapid attitude adjustment ($\kappa = 100$) leads to a more integrated society with

$I_{\text{int}}^t, v_{\text{out}}^t$ reaching non-zero values as $t \rightarrow \infty$, and with $\langle x_i^t \rangle_{\text{guest}} \rightarrow 0^-$ and $\langle x_i^t \rangle_{\text{host}} \rightarrow 0^+$. All these are hallmarks of a well-mixed, functional society, where guests and hosts share links, their socioeconomic progress is intertwined, and groups are not hostile to each other. On the other hand, slower attitude adjustment ($\kappa = 1000$) leads to a segregated society, where $I_{\text{int}}^t \rightarrow 0, v_{\text{out}}^t \rightarrow 0$, and where $\langle x_i^t \rangle_{\text{guest}}$ and $\langle x_i^t \rangle_{\text{host}}$ converge to non zero values as $t \rightarrow \infty$. In this case, there are no links connecting guests and nodes, there is no shared socioeconomic interest, and groups are hostile to each other. Society is fragmented and parallel societies have emerged. Note that these two opposite outcomes emerge from the same set of parameters, with the exception of κ .

Because of their superior number, it is the attitudes of hosts in particular that play a fundamental role in determining whether a society is segregated or not. This is consistent with findings from several surveys and societal observations [34, 40]. Recall that our initial conditions were set at $x_{i,\text{host}} = 1$, the most inhospitable. Figs. 4a and 4b show that this hostile environment drives the immigrant population towards segregation, unless attitudes can easily change, *i.e.* for small κ .

Results for $N_g = 20$ confirm the above scenario, with a small difference. Here, $I_{\text{int}}^t \rightarrow 0, v_{\text{out}}^t \rightarrow 0$ for both values of $\kappa = 100, 1000$ as $t \rightarrow \infty$, while $\langle x_i^t \rangle_{\text{guest}} \rightarrow 0^-$ and $\langle x_i^t \rangle_{\text{host}}$ converge to values that deviate only slightly from unity. In this case, the very few guests must initially interact with the many hosts and their attitude will become cooperative. Hosts on the other hand will not necessarily link to guests, and due to their numerical superiority can remain hostile towards them. Over time, separated enclaves of hosts and guests will emerge, with guests keeping their cooperative attitude, but in isolation from hosts, while hosts will largely remain in the same state as at the onset of the adaptation process. In this case, in order for a more cooperative society to emerge the value of κ must be even smaller. We have verified this numerically, finding that for $N_g = 20, \kappa \lesssim 40$ in order for a more integrated society to emerge.

3.3. Initially hostile host attitudes drive immigrants into enclaves. The importance of initial attitudes is further examined in Fig. 5, where at $t = 0$ hosts are extremely hospitable and $x_{i,\text{host}}^0 = 0$. Initial guest attitudes remain uncooperative at $x_{i,\text{guest}}^0 = -1$. All other parameters are set as in Figs. 3 and 4. Curves in Fig. 5a and 5b arise from the full model and should be compared to their counterparts in Fig. 4a and 4c.

In Fig. 5a we plot I_{int}^t . As can be seen, guests and hosts are no longer completely segregated. At early times, I_{int}^t decreases due to network remodeling, however at intermediate times, guests become more cooperative so that $x_{i,\text{guest}}^t \rightarrow 0^-$ and $I_{\text{int}}^t \rightarrow 1$ for long times. The early decrease of I_{int}^t is more significant for $\kappa = 1000$, since slow attitude adjustment leads to ineffective cross-group links and network remodeling will induce segregation. The decrease of I_{int}^t is also relatively more significant for $N_g = 200$ than for $N_g = 20$ under the same value of κ . This is because a larger guest population, and a larger $\tau_g = \kappa N/N_h$ will more slowly evolve its initially hostile attitudes, allowing for segregation to cut cross-group, ineffective connections. In Fig. 5b, we plot v_{out}^t which increases at early times in all cases except for $N_g = 200$ and under slow adjustment $\kappa = 1000$. This is due to network remodeling. As discussed above, the slow attitude adjustment prompts nodes to seek in-group connections at early times; the guest population is large enough to allow for this leading to segregation with $I_{\text{int}}^t \simeq 0.5$ and an initially decreasing v_{out}^t

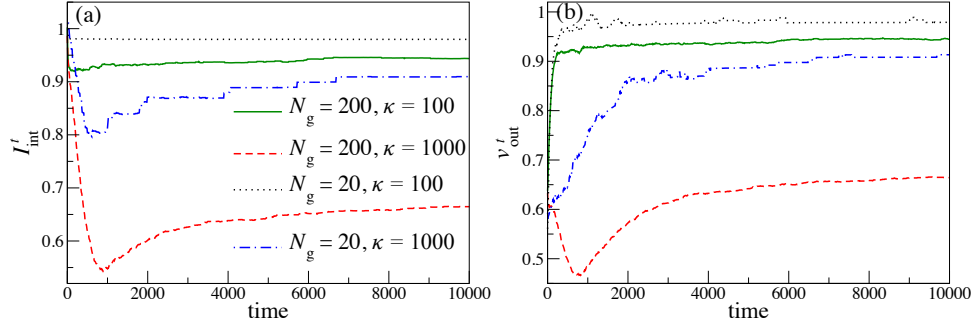


FIGURE 5. Dynamics of the integration index I_{int}^t in panel (a) and of the out-group reward fraction v_{out}^t in panel (b) for initially cooperative hosts. Parameters are the same as for the full model in Fig. 3, with initially cooperative hosts and uncooperative guests at $x_{i,\text{host}}^0 = 0^+$ and $x_{i,\text{guest}}^0 = -1$. (a) I_{int}^t decreases at the onset, eventually rising towards integration, where $I_{\text{int}}^t \rightarrow 1$ as $t \rightarrow \infty$. The initial decrease is more pronounced for slow attitude adjustment ($\kappa = 1000$) and for larger guest populations ($N_g = 200$) as described in the text. (b) v_{out}^t increases over long times as attitude adjustment allows for more cooperation between guests and hosts. Under slow attitude adjustment ($\kappa = 1000$) and large guest populations ($N_g = 200$), v_{out}^t decreases at the onset, with players seeking in-group connections. As guests and hosts become more cooperative v_{out}^t increases.

for the red-dashed curve. Due to the cooperative attitude of hosts however, guest eventually change their attitudes so that $x_{i,\text{guest}}^t \rightarrow 0^-$, and I_{int}^t and v_{out}^t increase.

One interesting finding is that when initial host attitudes are hostile, as shown in Figs. 4c and 4d, a larger guest population more effectively drives host attitudes towards cooperation. In contrast, when initial host attitudes are hospitable, as shown in Fig. 5a, a larger guest population results in less integration. In this case, the larger guest population is more resistant to attitude changes, and segregation may more easily emerge.

3.4. Higher initial connectivity facilitates better integration. The initial social connections assigned to migrants upon arrival may affect integration outcomes. As discussed in Section 2.4, one ideal scenario is that of welcoming programs that provide guests with prearranged social connections to hosts ($I_{\text{int}}^0 = 1$), another is that of completely isolated guests arriving in an already connected native society ($I_{\text{int}}^0 = 0$). In previous sections we only implemented these two extremes, perfect connectivity or total isolation. In this section we will consider more realistic, intermediate levels of initial guest connectivity.

Figure 6 illustrates the effects of three initial configurations: well connected guests, $I_{\text{int}}^0 = 0.91$ (blue-solid curve), intermediately connected guests, $I_{\text{int}}^0 = 0.37$ (green-dashed curve), and poorly connected guests $I_{\text{int}}^0 = 0.06$ (red-dotted curve). Initial attitudes are uncooperative, $x_{i,\text{host}}^0 = 1$ and $x_{i,\text{guest}}^0 = -1$. All model parameters are the same as in Fig. 3 with $N_g = 200$ and $\kappa = 100$.

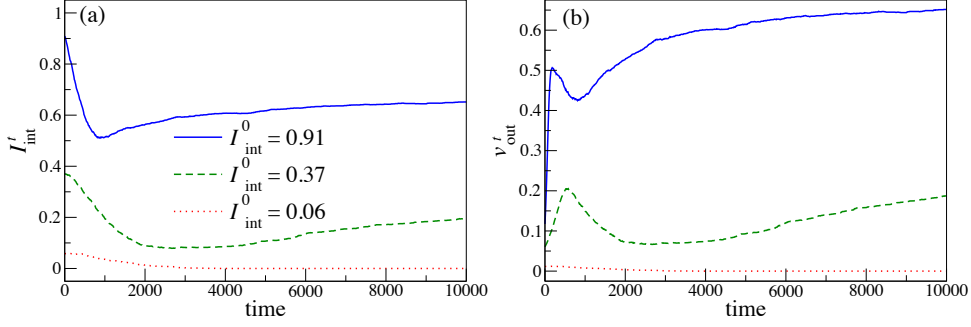


FIGURE 6. Dynamics of the integration index I_{out}^t in panel (a) and of the out-group reward fraction v_{out}^t in panel (b) under different initial random connectivities. Parameters are the same as in Fig 3 with initial hostile attitudes $x_{i,\text{host}}^0 = 1$ and $x_{i,\text{guest}}^0 = -1$. In the blue-solid curve $I_{\text{int}}^0 = 0.91$; in the green-dashed curve $I_{\text{int}}^0 = 0.37$; in the red-dotted curve $I_{\text{int}}^0 = 0.06$. (a) For all three cases, I_{int}^t decreases from the initial values, but only the initially poorly connected case of $I_{\text{int}}^0 = 0.06$ leads to full segregation, indicated by $I_{\text{int}}^t \rightarrow 0$ as $t \rightarrow \infty$. For the other two cases, $I_{\text{int}}^t \rightarrow 1$. (b) For all three cases v_{out}^t increases at the onset due to attitude adjustment, and later decreases due to network remodeling. Only $I_{\text{int}}^0 = 0.06$ leads to long-time $v_{\text{out}}^t \rightarrow 0$: as guest-host connections are severed, no socioeconomic utility can be shared. For the other two cases, v_{out}^t increases at long times, suggesting increasing rewards through cross-group connections.

The time evolution of I_{int}^t for all cases is shown in Fig. 6a. Here, I_{int}^t decreases at early times until guests and hosts begin adopting more cooperative attitudes. For the initially well connected case (blue-solid curve), I_{int}^t drops to $I_{\text{int}} \simeq 0.5$ before the trend is reversed at $t \sim 1000$. For the initially intermediately connected case (green-dashed curve), the decreasing trend is not reversed until $t \simeq 2500$ when $I_{\text{int}}^t \simeq 0.1$. Finally, for the initially poorly connected case (red-dotted curve) attitude adjustment cannot give rise to cooperation before $I_{\text{int}}^t \rightarrow 0$ and host and guest communities are fully segregated. Mirroring trends are seen in Fig. 6b where we plot the out-group reward fraction v_{out}^t . When guests are poorly connected at the onset (red-dotted curve), few links exist through which attitudes can change, guests become progressively segregated, and very little socioeconomic activity is shared. Hence, $v_{\text{out}}^t \rightarrow 0$ throughout. For the other two cases when there is more initial connectivity at the onset v_{out}^t increases at early times (blue-solid and red-dotted curves) as guests adopt cooperative attitudes ($x_{i,\text{guest}}^t \rightarrow 0^-$) through these initial guest-host connections. Later, network remodeling causes v_{out}^t to decline as cross-group connections are replaced with in-group ones. At longer times, host attitudes also evolve towards cooperation ($x_{i,\text{host}}^t \rightarrow 0^+$) from residual guest-host connections. Here, network remodeling no longer favors in-group connections, and v_{out}^t increases once more.

Although these results point to the importance of an initial network of connections for immigrants, in reality very few of them will have a support system upon

arrival. Many host countries may not have adequate resources or programs to foster such contact, and host and guest communities may view each other with suspicion. In the rest of this paper we attempt to identify best practices leading to integration, and look at how results vary depending on model parameters. We will consider a realistic, worst case initial condition: that of an initially equilibrated host community and a totally isolated guest cohort, as outlined in Section 2.4.

3.5. Dependence on parameters of cross-group reward, attitude adjustment rate, and sensitivity to attitude difference. We now study how results from the model defined in Eqs. 1–3 depend on its main parameters α , A_{in} , A_{out} , σ , and κ . In earlier sections, we set $A_{\text{in}} = A_{\text{out}} = A$ and determined analytically that the utility reaches a maximum $U_i^{\text{max}} = \alpha A[\ln(\alpha A) - 1]$ if each node has $m_{\text{opt}} = \alpha \ln(\alpha A)$ links. We have also verified this numerically for several α , A parameter choices. Note that setting $\alpha \lesssim A^{-1/2}$ leads to $m_{\text{opt}} \lesssim 1$ indicating a network with no links, which we have verified numerically. We also briefly discussed how κ affects the dynamics by comparing results from high ($\kappa = 1000$) and a low ($\kappa = 100$) regimes. Here we will conduct a more thorough investigation of the relevant parameters.

First we examine a scenario where $A_{\text{out}} \neq A_{\text{in}}$ and the effects of varying $A_{\text{out}}/A_{\text{in}}$ while keeping other parameters fixed. In Fig. 7a we plot the steady-state integration index $\langle I_{\text{int}}^* \rangle$ as a function of $A_{\text{out}}/A_{\text{in}}$, with $A_{\text{in}} = 10$, $\kappa \rightarrow \infty$, $\alpha = 3$, and $\sigma = 1$ for $N_{\text{g}} = 200$ guests and a total population of $N = 2000$ nodes, corresponding to $N/N_{\text{h}} = 1.11$. Note that setting $\kappa \rightarrow \infty$ is equivalent to activating network remodeling only, since the timescale for attitude change diverges, hence attitudes x_i^t will remain fixed at their initial values throughout the entire course of the dynamics. We also use two different initial conditions of total guest isolation but different initial attitudes. The blue-solid triangles represent initially cooperative populations with $x_{i,\text{host}}^0 = x_{i,\text{guest}}^0 = 0$, while the red-solid circles represent initially hostile populations with $x_{i,\text{host}}^0 = 1$, and $x_{i,\text{guest}}^0 = -1$. In both cases, guests have no connections at $t = 0$.

Each data point and relative error bar in Fig. 7a represents the mean and variance over 20 realizations, respectively. For the cooperative case (blue-solid triangles) as long as $A_{\text{out}}/A_{\text{in}} \lesssim 1$ in-group connections yield higher rewards and are preferred; hence the two populations are almost completely segregated and $\langle I_{\text{int}}^* \rangle \rightarrow 0.1$. Conversely, when $A_{\text{out}}/A_{\text{in}} \gtrsim 1$ out-group connections are preferred, and $\langle I_{\text{int}}^* \rangle \rightarrow N/N_{\text{h}} = 1.11$, indicating reverse segregation. When $A_{\text{out}}/A_{\text{in}} \simeq 1$ out-group and in-group connections are equivalent in terms of their socioeconomic weight and integration is observed at $\langle I_{\text{int}}^* \rangle \rightarrow 1$. Note the sharp transitions between regimes. The progression segregation \rightarrow integration \rightarrow reverse segregation as a function of $A_{\text{out}}/A_{\text{in}}$ also appears for the uncooperative conditions (red-solid circles). However, in this case transitions are shifted towards the right, indicating that out-group connections must yield higher socioeconomic gain to promote integration (or reverse segregation) in order to overcome the initial hostility among players. Here, segregation persists until $A_{\text{out}}/A_{\text{in}} \lesssim 6$ for which $\langle I_{\text{int}}^* \rangle \lesssim 0.1$, full integration $\langle I_{\text{int}}^* \rangle \rightarrow 1$ arises for $6 \lesssim A_{\text{out}}/A_{\text{in}} \lesssim 8$ and reverse segregation at $\langle I_{\text{int}}^* \rangle \rightarrow N/N_{\text{h}} = 1.11$ appears only for $A_{\text{out}}/A_{\text{in}} \gtrsim 8$. Note that in both cases since attitudes are fixed, rewards are given by $u_{ij} = A_{\text{out}} e^{-2}$ if through out-group connections, and by $u_{ij} = A_{\text{in}}$ if through in-group ones. The two will be the same for $A_{\text{out}}/A_{\text{in}} = e^2 = 7.39$.

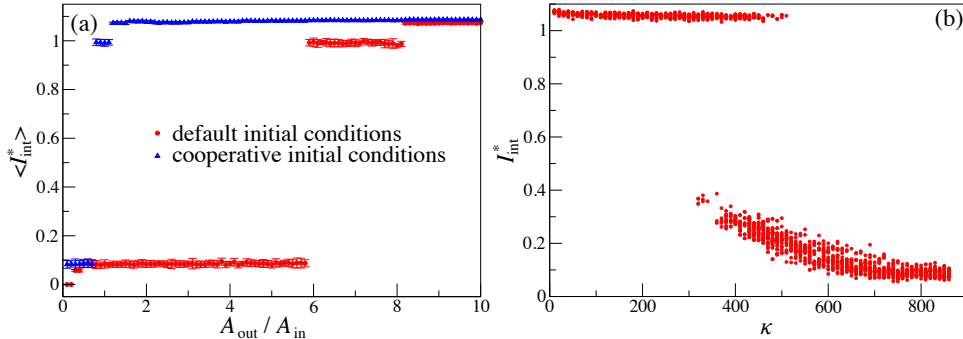


FIGURE 7. Integration index at steady state. In panel (a) $\langle I_{\text{int}}^* \rangle$ is averaged over 20 realizations and plotted as a function of $A_{\text{out}}/A_{\text{in}}$ with $\kappa = \infty$. The bar indicates the variance. In panel (b) single representations I_{int}^* are shown as a function of κ with $A_{\text{out}}/A_{\text{in}} = 2$. Other parameters are set at $\alpha = 3$ and $\sigma = 1$, with $N_{\text{h}} = 1800$ and $N_{\text{g}} = 200$. In both panels red solid circles represent initially unconnected, hostile hosts and guests, $x_{i,\text{host}}^0 = 1$, $x_{i,\text{guest}}^0 = -1$; blue triangles correspond to fully cooperative initial conditions $x_{i,\text{host}}^0 = x_{i,\text{guest}}^0 = 0$. When the ratio $A_{\text{out}}/A_{\text{in}}$ increases, the long-time state of the network changes from segregation to uniform mixture, and finally to reversed segregation. The transition for the default initial conditions occurs at larger $A_{\text{out}}/A_{\text{in}}$ ratios, compared to the cooperative initial conditions, as the former require higher compensation from out-group connections to overlook the hostile attitudes between guests and hosts. In panel (b) each data point corresponds to one realization. Increasing attitude adjustment time scale κ leads to increased likelihood of segregation. A bimodal regime emerges for intermediate κ .

These results indicate that to promote integration, cross-group connections must generate higher rewards than in-group ones. This may be realized, for example, if the immigrant population possesses skill sets that complement those of the host population. Since no attitude adjustment is allowed in the dynamics, Fig. 7a suggests that integration may occur even if groups maintain their hostility towards each other as long as the socioeconomic rewards are large enough, as seen for the uncooperative case (red-solid circles). Finally, note that the same parameter sets yields very different results for a wide range of $A_{\text{out}}/A_{\text{in}}$ values, as can be seen by the bimodal values of $\langle I_{\text{int}}^* \rangle$ in Fig. 7a and underlying the role of initial conditions in determining integration or segregation.

We examine the effects of varying κ in Fig. 7b. Here, we use the same parameters as in Fig. 7a with $A_{\text{in}} = 10$, $A_{\text{out}} = 20$, $\alpha = 3$, and $\sigma = 1$. The ratio $A_{\text{out}}/A_{\text{in}} = 2$ provides modest incentives for guests and hosts to collaborate. We consider initially hostile guests and hosts at $x_{i,\text{host}}^0 = 1$ and $x_{i,\text{guest}}^0 = -1$, and omit fully cooperative initial conditions $x_{i,\text{host}}^0 = x_{i,\text{guest}}^0 = 0$ since, in this case, changes to κ will not alter the dynamics. Each red solid circle in Fig. 7b is the result of a single simulation; for each value of κ simulations are repeated 20 times. The dot-plot shows that if attitude adjustment is sufficiently fast ($\kappa \lesssim 300$) reverse segregation arises and

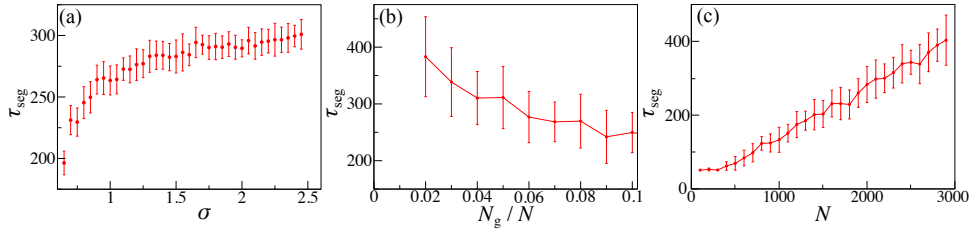


FIGURE 8. Time τ_{seg} to reach $\langle I_{\text{int}}^* \rangle = 0.1$, where 90% of guest nodes are segregated as a function of (a) the sensitivity to the reward function σ , (b) the relative guest population N_g/N and (c) the total population N assuming $N_g = 0.1N$. Other parameters are set to $\alpha = 3$, $A_{\text{in}} = A_{\text{out}} = 10$, $\kappa = 600$ in all panels. In panel (a) $N_g = 200$, $N = 2000$; in panel (b) $\sigma = 1$ and $N = 2000$; in panel (c) $\sigma = 1$. In all three cases, guests and hosts are initially unconnected and hostile to each other, $x_{i,\text{host}}^0 = 1$ and $x_{i,\text{guest}}^0 = -1$. Each data point and its error bar represent the mean and the variance over 20 simulations. In panel (a) increasing σ allows for more tolerance to attitude differences, increasing the time to segregation. In panel (b) the higher guest population ratio leads to faster segregation as guests are more likely to establish in-group connections, forming guest only enclaves. In panel (c) the time to segregation increases with the overall population, for a constant 10% guest population.

$I_{\text{int}}^* \simeq N/N_h = 1.11$; guests and hosts adopt cooperative attitudes before segregation can arise. For very slow attitude adjustments ($\kappa \gtrsim 550$), almost complete segregation as $I_{\text{int}}^* \rightarrow 0.1$ is the only outcome. A bimodal regime instead arises for intermediate values of $300 \lesssim \kappa \lesssim 550$ where segregation and reverse segregation are both likely. The bimodal feature of Fig. 7b is indicative of the different timescales between the two competing mechanisms of network remodeling and attitude adjustment. If attitude adjustment is fast compared to network remodeling (low κ) guests will quickly adopt cooperative attitudes, and guest-only enclaves will not be formed. Conversely, if attitude adjustment is slow compared to network remodeling (large κ) guest-only enclaves will form hindering cooperativity. In between these limits, is a regime where the timescales of network remodeling and attitude adjustment are comparable, and the outcomes stochastic.

The last parameter we examine here is σ , which regulates the sensitivity of the reward function u_{ij}^t to attitude differences $|x_i^t - x_j^t|$ in Eq. 1. Note that $\sigma \rightarrow \infty$ renders u_{ij}^t independent of $|x_i^t - x_j^t|$. Finite values of σ , however large, do not determine whether in-group or out-group connections are preferred. This parameter thus will only affect the timescale of the dynamics. In particular, since larger values of σ attenuate the sensitivity of u_{ij}^t to $|x_i^t - x_j^t|$ we expect larger values of σ to also be associated with slower dynamics. We have verified this by considering the time to reach 90% segregation, defined as $I_{\text{int}}^t = 0.1$, as a function of σ and for a variety of parameter choices. In Fig. 8a we plot the time to segregation, denoted by $\tau_{\text{seg}}(\sigma)$, for the particular case of $\alpha = 3$, $A_{\text{in}} = A_{\text{out}} = 10$, and $\kappa = 600$, with initially hostile populations $x_{i,\text{host}}^0 = 1$ and $x_{i,\text{guest}}^0 = -1$ and no initial link between hosts and guests. As can be seen, $\tau_{\text{seg}}(\sigma)$ increases with σ . This result suggests that

decreasing the sensitivity to attitude differences, particularly between guests and hosts, results in longer times to full segregation. This larger time window between migrant arrival and full segregation may provide better opportunities to implement interim policies that promote cooperation.

3.6. High immigrant ratios and small native populations promote segregation. In this section we examine the effects of migrant population sizes compared to that of the native community. We are particularly interested in the uncooperative, segregated case and examine how the time to segregation τ_{seg} depends on the fraction of guests. Under parameters and conditions that favor segregation, we expect larger guest populations will more quickly evolve to the uncooperative steady state. We thus consider a scenario where at steady state guests segregate, resulting in $x_{i,\text{guest}}^t \rightarrow -1$, $x_{i,\text{host}}^t \rightarrow 1$, $I_{\text{int}}^t \rightarrow 0$ as $t \rightarrow \infty$. We then keep all parameters fixed, including the total population N , and modify only N_g to study τ_{seg} as a function of the N_g/N ratio. In Fig. 8b we show $\tau_{\text{seg}}(N_g/N)$ for the representative case of $\alpha = 3$, $A_{\text{in}} = A_{\text{out}} = 10$, $\sigma = 1$, and $\kappa = 600$. Initial conditions are initially hostile populations $x_{i,\text{host}}^0 = 1$ and $x_{i,\text{guest}}^0 = -1$ and no initial link between hosts and guests. As can be seen, $\tau_{\text{seg}}(N_g/N)$ is a decreasing function of its argument, as expected. Here, attitude adjustment timescales κ are not affected by N_g/N , however a larger guest population makes in-group interactions more likely under the dynamics specified in Section 2.3. The increased guest-guest pairing allows for uncooperative attitudes to be maintained for longer times, lowering the utility reward from cross-group interactions and hastening the severing of such links. Numerically lower guest populations instead carry a higher likelihood of interacting with hosts, fostering cooperative attitudes for longer times, and allowing for socioeconomically advantageous cross-group connections to emerge. Several sociological reports show that conflicts between a majority N_h and a minority N_g population are less intense and frequent, if the majority population greatly exceeds that of the minority, $N_h \gg N_g$ [2]. We can conjecture that such conflicts arise when hosts and guests are extremely polarized and segregated from each other, as for the case illustrated above. Our results show that as N_g/N increases segregation, and by proxy, the emergence of conflict between the two groups increases as well, confirming these sociological findings.

Finally, in Fig. 8c we plot $\tau_{\text{seg}}(N)$ as a function of the total population N by fixing $N_g = 0.1N$. All other parameters and initial conditions are the same as in Fig. 8b. As can be seen, larger N populations lead to longer times to segregation $\tau_{\text{seg}}(N)$. This result implies that the same fraction of migrants can be more easily accommodated in larger communities.

3.7. Transitioning from segregation to integration. In this section we study the interplay between the two timescales, κ and τ_{seg} , that determine whether or not guest-only enclaves will form, starting from an initially hostile and unconnected mixture of guests and hosts. In Fig. 7b we showed that fast attitude adjustment (small κ) prevents the formation of guest-only enclaves if incentives are in place to support cross-group collaborations ($A_{\text{out}}/A_{\text{in}} > 1$). As shown in Fig. 8b, increasing the guest population ratio N_g/N , shortens the time to segregation τ_{seg} and facilitates the establishment of guest-only enclaves.

To study the interplay between κ and τ_{seg} we plot $\langle I_{\text{int}}^* \rangle$ in Fig. 9a as a function of κ and N_g/N for the representative case of $\alpha = 3$, $A_{\text{in}} = 10$, $A_{\text{out}} = 20$, $\sigma = 1$, and $N = 2000$. The populations are initiated with hostile attitudes $x_{i,\text{host}}^0 = 1$ and

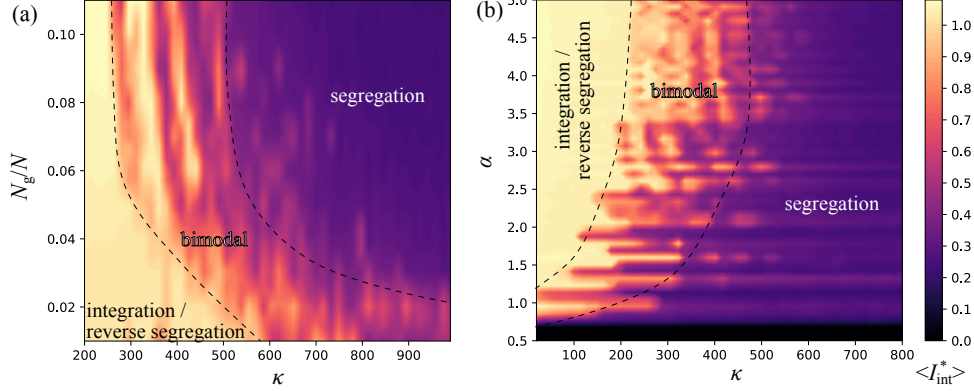


FIGURE 9. Integration index at steady state. $\langle I_{\text{int}}^* \rangle$ is averaged over 10 realizations and plotted as a function of κ and N_g/N with $\alpha = 3$ in panel (a), and as a function of κ and α with $N_g/N = 0.1$ in panel (b). Other parameters are set at $A_{\text{in}} = 10$, $A_{\text{out}} = 20$, $\sigma = 1$, and $N = 2000$. In both panels guests and hosts are initially unconnected, with hostile attitudes, $x_{i,\text{host}}^0 = 1$, $x_{i,\text{guest}}^0 = -1$. In panel (a), for smaller N_g/N , the transition from segregation to integration (or reverse segregation) occurs at larger κ . In panel (b) increasing α causes the transition point to shift towards larger κ .

$x_{i,\text{guest}}^0 = -1$, and no cross-group initial link. As can be seen, decreasing κ induces a transition from segregation at $\langle I_{\text{int}}^* \rangle \rightarrow 0$ for large κ , to integration at $\langle I_{\text{int}}^* \rangle \rightarrow 1$, for small κ , or even reverse segregation at $\langle I_{\text{int}}^* \rangle \rightarrow N/N_h$, for very small κ . Transitions towards integration thus are favored in societies where attitudes towards the other are less entrenched and where guests and hosts more readily adapt to each other. Fig. 9a also shows that transitions depend on the value of N_g/N , and indirectly on τ_g : larger values of N_g/N imply shorter transition κ values. This is because increases in N_g/N , and consequently decreases in τ_{seg} , correspond to less time for attitude adjustment to affect cross-group utility gains. Larger percentages of migrants N_g/N imply that individual attitudes κ must be even more open to diversity if one is to observe the same target integration index $\langle I_{\text{int}}^* \rangle$. Note that in Fig. 9a we can also identify a bimodal regime, where $\langle I_{\text{int}}^* \rangle$ takes on values between zero and N/N_h where final integration outcomes depend on stochastic events.

Finally, in Fig. 9b, we study how the integration index $\langle I_{\text{int}}^* \rangle$ depends on κ and α , the latter controlling the average number of connections associated with each node. We fix $N_g = 200$ and use the same parameter values and initial conditions as in Fig. 9a. Increasing α corresponds to increasing the number of connections per node. As can be seen in Fig. 9b the same progression seen in Fig. 9a of transitioning from segregation to integration can be seen upon lowering κ for fixed α . Increasing α leads these transition points to shift towards larger values of κ , signifying that more connections per node allow for slower attitude adjustment to achieve the same integration value $\langle I_{\text{int}}^* \rangle$. Beyond $\alpha \gtrsim 3$ however, the transition regime of κ appears not to change appreciably, implying little sensitivity of $\langle I_{\text{int}}^* \rangle$ to the average number of connections per node.

4. Discussion and conclusions. As recent news reports and historical analysis attest, societal dynamics after the influx of newcomers depends on many factors, including the socioeconomic environment of the host country, the adaptability of the immigrant population, the open-mindedness of natives, and the degree of compatibility between guest and host values. Our model is based on the assumption that upon resettlement immigrants have two primary goals: socioeconomic prosperity and social acceptance. Game-theoretic rules are used to model socioeconomic gains through a utility function to be maximized, leading to network remodeling. Attitude adjustment is instead driven by opinion dynamics rules. The two processes occur at different timescales: network remodeling at a timescale of unity, and attitude adjustment at a timescale of $\tau_g = \kappa N/N_h$ for guests. Due to their numerical superiority, hosts constitute a quasi-infinite bath: they greatly impact migrant dynamics, but their own characteristics change only marginally and over very long timescales, given by $\tau_h = \kappa N/N_g \gg \tau_g$.

The interplay between the various timescales is shown across our analysis. For low values of κ attitude adjustment is fast, cross-group socioeconomic gains are robust and immigrants are less likely to form segregated enclaves. For large values of κ , attitudes change very slowly, and the formation of isolated guest niches becomes the most efficient way for guests to advance their socioeconomic status. Our results are consistent with findings from public goods evolutionary game theory models where interactions among various social contexts, such as population diversity and cultural tolerance, lead to different ratios between the timescales for strategy evolution and network structure remodeling; such timescale difference determines whether cooperative patterns emerge [46, 48]. In particular, cooperators will outweigh defectors if strategy evolution is faster than network remodeling [43].

The socioeconomic reward structure associated with guest-host collaborations also plays an important role in determining societal outcomes. As shown in Fig. 7a larger out-group versus in-group rewards, represented by the $A_{\text{out}}/A_{\text{in}}$ ratio, are more conducive to integration. Out-group rewards promote the willingness among a mixed population to pursue conformity, which was identified as a key psychological factor for cooperative patterns to emerge in game theoretical models [31]. These results suggest that segregation may be avoided if newcomers carry inherent advantages, for example in the form of skill sets that are complementary to those of the native population, or if governmental incentives are established to promote cross-group interactions. Fig. 7a also reveals the fundamental role of initial conditions. If out-group rewards are much larger than in-group ones, $A_{\text{out}} \gg A_{\text{in}}$, cooperation arises regardless of initial conditions. However, if the two are comparable, Fig. 7a shows that integrated or segregated societies can emerge from the same parameter set, and that whether one configuration prevails over the other depends on the initial conditions. Of course, integration is the more likely outcome if the initial attitudes are highly cooperative, while segregation will typically emerge from initial scenarios where guests and hosts are highly hostile to each other. This finding is also consistent with sociological observations [34, 40] where the attitude of the majority population is identified as a primary determinant in minority segregation.

We also find that given the same social environment, a higher immigrant population ratio results in segregation, while a larger total population will more harmoniously absorb the same percentage of immigrants, which agrees with previous Ising-type sociophysical models of immigrant integration [24]. Our results suggest

accommodating newcomers in accordance with the host population. Small, possibly rural, communities may not be optimal conduits to integration compared to more populous cities, especially if the percentage of migrants is large. Examples of countries distributing refugees in proportion to the population of receiving municipalities include Denmark, from 1986 to 1998, Sweden, from 1987 to 1991, and the United Kingdom since 2000. However, refugees were later found to relocate to larger cities [13, 45, 51], attracted by the presence of more co-ethnics, job opportunities and housing. Recent studies have also observed higher segregation of immigrants in rural areas, especially when the size of the migrant group is large and hosts are hostile to guests [32, 36, 37].

Our model does not include spatial dependence or geographical factors in making and maintaining social connections. For example, the turnover rate of social connections may be higher in denser areas, leading to inhomogeneous timescales for attitude changes in the network. We also do not consider the effects of virtual connectivity, whereby internet connections may render spatial dependence less relevant while also accelerating segregation as finding co-cultural companions is facilitated in online venues.

Our model may be generalized by introducing a continuous influx of immigrants, instead of assuming a fixed initial guest population. A continuous influx may allow us to include in-group interactions between immigrants arriving at different times, and to study cooperation or antagonism among them. To further extend our model across generations, since earlier immigrants and their descendants eventually may be considered part of the native community, the $x_i^t = 0$ barrier between hosts and guests must be relaxed in order to allow for generational crossover between groups. Similarly, long-term attitude differences between hosts and guests can lead to open conflict or violence that may curb socioeconomic rewards, including in-group ones. This mechanism would require higher order corrections and feedback mechanisms that are currently not included in our work.

Moreover, our utility function U_i^t carries the same form for every node and penalizes those with too many connections. As a result, all nodes converge towards an average number of connections, which is not realistic, since actual social networks take on small-world characteristics, with hub nodes having a large number of connections [49]. Our model may be improved by introducing more nuanced utility functions. For example, we may postulate that nodes with larger socioeconomic utility are able to maintain a larger number of connections, compared to those with lower utility, creating a mechanism for hubs to emerge [21]. All these factors may influence the entire society and change host and guest perceptions, in a positive or negative way.

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