

Averages along cubes for not necessarily commuting m.p.t.

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Abstract

Averages along cubes have been introduced by T. Gowers in his proof of Szemerédi's theorem on the existence of arbitrary long arithmetic progressions in subset of integers with positive upper density. In this talk we look at their ergodic counterparts in the case of not necessarily commuting measure preserving transformations. More precisely we show that if (X, B, μ, T_i) are not necessarily measure preserving systems on the same finite measure space and if $f_i, 1 \leq i \leq 6$ are bounded functions then the averages

$$\frac{1}{N^3} \sum_{n,m,p=1}^N f_1(T_1^n x) f_2(T_2^m x) f_3(T_3^p x) f_4(T_4^{n+m} x) f_5(T_5^{n+p} x) f_6(T_6^{m+p} x)$$

converge almost everywhere. We will also show that extension of Khintchine's famous recurrence theorem can be obtained in the case of not necessarily commuting measure preserving transformations.