Assignment 3 Due October 24 Covers: Sections 2.1-2.3

- Q1. Do Exercise 1(cdfh) of Section 2.1 of the textbook.
- Q2. Do Exercise 7 of Section 2.1 of the textbook.
- Q3. Do Exercise 10 of Section 2.1 of the textbook.
- Q4*. Do Exercise 17 of Section 2.1 of the textbook.
- Q5. Do Exercise 1(bcdf) of Section 2.2 of the textbook.
- Q6. Do Exercise 2(aceg) of Section 2.2 of the textbook.
- Q7. Do Exercise 7 of Section 2.2 of the textbook.
- Q8. (a) Let V, W be vector spaces, and let $T : V \to W$ be a linear transformation. Let U be a subspace of W. Show that the set

  $$T^{-1}(U) := \{v \in V : T(v) \in U\}$$

  is a subspace of V. Explain why this shows that the null space $N(T)$ is also a subspace.
- (b) Let V, W be vector spaces, and let $T : V \to W$ be a linear transformation. Let X be a subspace of V. Show that the set

  $$T(X) := \{Tv : v \in X\}$$

  is a subspace of W. Explain why this shows that the range $R(T)$ is also a subspace.
- Q9*. Show, *without doing Gaussian elimination or any other computation*, that there must be a solution to the system

  $$\begin{array}{lllll} 12x_1 & +34x_2 & +56x_3 & +78x_4 & = 0 \\ 3x_1 & +6x_2 & +2x_3 & +10x_4 & = 0 \\ 43x_1 & +21x_2 & +98x_3 & +76x_4 & = 0 \end{array}$$

  such that the x_1, x_2, x_3, x_4 are not all equal to zero. [**Hint:** consider the linear transformation $T : \mathbf{R}^4 \to \mathbf{R}^3$ defined by

$$T(x_1, x_2, x_3, x_4) := (12x_1 + 34x_2 + 56x_3 + 78x_4,$$

$$3x_1 + 6x_2 + 2x_2 + 10x_4, 43x_1 + 21x_2 + 98x_3 + 76x_4).$$

What can you say about the rank and nullity of T?

- Q10. Find a *non-zero* vector $v \in \mathbf{R}^2$, and two ordered bases β, β' of $\mathbf{R}^2$, such that $[v]_\beta = [v]_{\beta'}$ but that $\beta \neq \beta'$.