Mathematics 245B Terence Tao Midterm, Feb 11, 2003

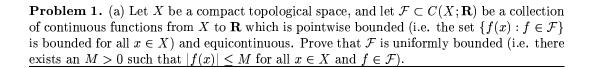
Instructions: Try to do all three problems; they are all of equal value. There is plenty of working space, and a blank page at the end.

You may enter in a nickname if you want your midterm score posted.

Good luck!

Name:	
Nickame:	
Student ID:	
Signature:	
G	
	Problem 1.
	Problem 2.
	Problem 3.

Total: ____



(b) Given an example to show that the results in part (a) fail if the assumption of equicontinuity is dropped.

Problem 2. (a) Let X, Y be topological spaces, and suppose that Y is Hausdorff. Let A be a dense subset of X, and let $f: X \to Y$ and $g: X \to Y$ be continuous functions. Show that if f and g agree on A (i.e. f(x) = g(x) for all $x \in A$), then they agree on all of X (i.e. f and g are identical).

⁽b) Now suppose that X is normal, and A is a subset of X which is *not* dense. Show that there exist continuous functions $f:X\to \mathbf{R}$ and $g:X\to \mathbf{R}$ which agree on A but do not agree on all of X.

Problem 3. Let X be a Banach space, and let $l^1(\mathbf{Z})$ be the space of absolutely summable sequences $(x_n)_{n \in \mathbf{Z}}$, with the l^1 norm $\|(x_n)_{n \in \mathbf{Z}}\|_{l^1(\mathbf{Z})} := \sum_{n \in \mathbf{Z}} |x_n|$. For each integer n, let $e_n \in l^1(\mathbf{Z})$ be the element of $l^1(\mathbf{Z})$ whose n^{th} entry is 1 and whose other entries are zero (thus $(e_n)_m = 1$ if m = n and $(e_n)_m = 0$ otherwise).

Let $T: l^1(\mathbf{Z}) \to X$ be a linear transformation. Show that T is continuous if and only if the set $\{T(e_n): n \in \mathbf{Z}\}$ is a bounded subset of X.