Midterm 2

Last Name: ________________________________
First Name: ________________________________
Student ID: ________________________________
Signature: ________________________________

Section: Tuesday: Thursday:
3A       3B       TA: Ioannis Lagkas-Nikolos
3C       3D       TA: Fei Xie
3E       3F       TA: Sangchul Lee

Instructions: Do not open this exam until instructed to do so. You will have 50 minutes to complete the exam. Please print your name and student ID number above, and circle the number of your discussion section. You may not use calculators, books, notes, or any other material to help you. Please make sure your phone is silenced and stowed where you cannot see it. You may use any available space on the exam for scratch work. If you need more scratch paper, please ask one of the proctors. You must show your work to receive credit. Please circle or box your final answers.

Please do not write below this line.

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1. (10 pts) Suppose a wire is wound along a cone in such a way that its path is described by

\[ \vec{r}(t) = \langle t^2, t \cos t, t \sin t \rangle, \quad 1 \leq t \leq 4 \]

If the charge density (per unit length) along the wire is given by

\[ \rho(x, y, z) = \frac{y^2 + z^2}{\sqrt{x}} \]

find the total charge in the wire.
2. (10 pts) Let $C$ be a helix of radius 3, centered along the $z$-axis, that makes 17 complete rotations clockwise while going up 4 units in the $z$ direction, starting from the point $(0, 3, 1)$. Compute the following line integral:

$$\int_C \sin y \, dx + \left( x \cos y - \frac{1}{z} \right) \, dy + \frac{y}{z^2} \, dz.$$ 

(Hint: You don’t need to parametrize $C$.)
3. (10 pts) Suppose \( \vec{F}(x, y) = (F_1(x, y), F_2(x, y)) \) is a vector field that is defined and differentiable everywhere in the \( xy \)-plane except at the points \((1, 0)\) and \((-1, 0)\). Suppose you know that, at every point in its domain,

\[
\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}
\]

(a) Let \( C \) be the circle of radius 1 centered at the point \((0, 1)\), oriented clockwise. Compute \( \oint_C \vec{F} \cdot d\vec{s} \). Justify your answer!

(b) Is \( \vec{F} \) conservative on the domain \( D \) shown to the right? Explain briefly.

YES

NO

NOT ENOUGH INFO

(c) Is \( \vec{F} \) conservative on its whole domain?

\( \text{Explain briefly.} \)

YES

NO

NOT ENOUGH INFO
4. (10 pts) Let $S$ be the portion of the surface $x^2 + y^2 - z^2 = 1$ where $0 \leq z \leq 3$, oriented with upward-pointing normal vectors. Compute the flux of the vector field

$$\vec{F}(x, y, z) = \langle xz, yz, z^2 \rangle$$

across the surface $S$. 