(1) Consider the equation \((x - y)^2 = x + y - 2\). Find the tangent line to the graph of this equation at the point \((1, 1)\).

(2) Consider the graph of the equation \(x^{2/3} + y^{2/3} = 8\).
   
   (a) Find \(dy/dx\)
   
   (b) Find all points on the graph where the tangent line has slope \(-1\).

(3) Find the derivative of the following functions:

(a) \(f(x) = x \sin(x^2)\)

(b) \(f(x) = 2^{2x+1}\)

(c) \(f(x) = (3x + \pi)^{100}\)

(d) \(f(x) = e^{\pi/3}\)

(e) \(f(x) = \cos(2x) + \cos(\pi/5)\)

(f) \(f(x) = \frac{x^2+1}{\sqrt{x-1}}\)

(g) \(f(x) = \cos(x^2 + 2x) + \ln(3x)\)

(4) Calculate the following limits (if they exist):

(a) \(\lim_{x \to 1} \ln(x) \cos(\pi x)\)

(b) \(\lim_{x \to 2} \frac{\sin(x) + 2}{\cos(x^2-4)}\)

(c) \(\lim_{x \to \infty} \frac{x^3-1}{x-2x^3}\)

(d) \(\lim_{x \to 0^+} \ln(x)\)

(e) \(\lim_{x \to 3} \frac{x^2-2x+3}{x^3-9}\)

(f) \(\lim_{x \to 0} \frac{\sin(x^2)}{2x^2}\)

(g) \(\lim_{x \to 0} \frac{1-\cos(x^3)}{3x}\)

(h) \(\lim_{x \to \infty} \frac{e^{x+1}}{x^2+3}\)

(i) \(\lim_{x \to 0} \frac{a^x-b^x}{x^2-x}\), where \(a, b > 0\)

(5) Sketch the graphs of the following functions. Find where \(f(x)\) is increasing/decreasing, and mark any inflection points, and any horizontal or vertical asymptotes.

(a) \(f(x) = x^4 - 4x\)

(b) \(f(x) = x + \frac{1}{x}\)

(c) \(f(x) = x - 3x^{1/3}\)

(d) \(f(x) = \frac{3x}{x-1}\)

(e) \(f(x) = \frac{x^2+4}{x^2+3}\)

(f) \(f(x) = e^{-x^2/2}\)
Figure 1: Problem 6

Figure 2: Problem 7
(6) Figure 1 shows three graphs. One is \( f(x) \), one is \( f'(x) \), and one is \( f''(x) \). Determine which is which.

(7) Consider the graph of \( y = f(x) \) in figure 2. List \( f(-1) \), \( f'(-1) \), and \( f''(-1) \) in increasing order.

(8) Calculate \( 16^{3/4} \) and use a linear approximation to estimate \( 17^{3/4} \).

(9) A population has been increasing linearly since 1990. In 2000, the population was 100, and in 2005, the population was 500. Find an equation for \( P(t) \), where \( P \) is the population and \( t \) is the number of years since 1990, and use it to approximate the population in 2007.

(10) You have 500 grams of a radioactive substance whose half-life is 20 days. Let \( N(t) \) denote the amount of the substance left after \( t \) days.

(a) Find a formula for \( N(t) \).

(b) Find how long it will take for 80% of the substance to decay.

(11) Bill is studying the relationship between the expected lifespan (\( S \)) of a new organism and its length (\( L \)). The length is measured in cm and the lifespan is in minutes. Bill predicts that the lifespan is proportional to the \( n \)th power of its length, where \( n > 0 \) is some number.

(a) Show that the elasticity of the lifespan does not depend on the proportionality constant or the length, but that it does depend on \( n \).

(b) If doubling the length increases the lifespan 16-fold, find \( n \).

(c) Bill finds that an organism with length 2 cm has a lifespan of one hour and four minutes. If the length is increased by .01 cm, estimate how much longer the lifespan would be.

(d) Bill wants to measure the lifespan of the organism to within .04% of its true value. Within what percent error does Bill need to measure its length to guarantee this accuracy?
Answers:

(1) \( y - 1 = -1(x - 1) \)

(2) (a) \( \frac{dy}{dx} = -\frac{1}{3}x^{1/3} \)
(b) \((8,8)\) and \((-8,-8)\).

(3) (a) \( 2x^2 \cos(x^2) + \sin(x^2) \)
(b) \( 2 \cdot 2^{2x+1} \ln(2) \)
(c) \( 300(3x + \pi)^{99} \)
(d) 0
(e) \(-2\sin(2x) \)
(f) \( \frac{2x\sqrt{x^2-1}+x^2+1}{x^2-1} \)
(g) \(-2x + 2 \)

(4) (a) 0
(b) 4
(c) \(-1/2 \)
(d) DNE (or \(-\infty\))
(e) 2/3
(f) 1/3
(g) 0
(i) 0
(j) \(-\ln(a) - \ln(b)\)

(5) Check using Wolfram Alpha or a graphing calculator.

(6) Red is \( f(x) \), blue is \( f'(x) \), and green is \( f''(x) \)

(7) \( f'(-1) < f(-1) < f''(-1) \)

(8) \( 16^{3/4} = 8 \) and \( 17^{3/4} \approx 67/8 \)

(9) \( P(t) = 100 + 80(t - 10) \), and the population in 2007 is approximately 660.

(10) \( N(t) = 500(.5)^{t/20} \) and it will take \( 20 \ln(.2)/\ln(.5) \) days for 80% to decay.

(11) (a) Elasticity is \( n \), which does not depend on \( k \) (the constant), or \( L \).
(b) \( n = 4 \)
(c) 1.28 minutes.
(d) .01%. 