(1) Which of the following functions are power functions? Explain your reasoning.

(a) \( f(x) = 2x^2 - 1 \)
(b) \( f(x) = (2x^2)^3 \)
(c) \( f(x) = x^{3x} \)
(d) \( f(x) = 2 \cdot 3^x \)
(e) \( f(x) = \frac{x^2}{3\sqrt{x}} \)

(2) Scientists studying a certain species found that the length \( L \) (in feet) and weight \( W \) (in pounds) was related by \( L = aW^b \), where \( a \) and \( b \) are numbers.

(a) If given various data points \((W, L)\), to use a linear regression, would you consider a semi-log plot or a log-log plot?

(b) Suppose the following points \((W, L)\) are given: \((1, 2.3), (2, 9.2), (3, 20.7), (4, 36.8)\), draw either a semi-log plot or a log-log plot (whichever one you chose in (a)).

(c) Find \( a \) and \( b \) using your plot.

(3) Sketch the graphs of the following functions (Hint: Start with a “parent” graph and use graph transformations)

(a) \( f(x) = 1 - e^{-x} \)
(b) \( f(x) = (x - 2)^2 + 3 \)
(c) \( f(x) = \sqrt{x} - 2 \)
(d) \( f(x) = 2\cos(3x) - 1 \)
(e) \( f(x) = \frac{1}{x^2} + 4 \)
(f) \( f(x) = \ln(1 - x) \)

(4) Find an exponential function going through the points \((0, 3)\) and \((2, 12)\).

(5) Suppose you put $500 dollars into an account which gives 20% annual interest.

(a) Find the amount in the account after 1 year. After 2 years? After \( t \) years?

(b) Repeat part (a) under the assumption that the interest is compounded twice a year.

(c) How much money is in the account after \( t \) years if the interest is compounded \( n \) times a year.

(d) As \( n \) gets larger and larger, the amount of money in the account after 1 year tends to a specific number. Find that number.

(6) Evaluate:

(a) \( \log_2(1/4) \)
(b) \( \log_5(125) \)

(7) Solve for \( x \):
(a) $2 = e^{x+2}$
(b) $2 = 4 \ln(2x) - 2$
(c) $3 - \log_2(x) = 5$

(8) The half life of a certain substance is 2 days. If you start out with 100 grams of the substance:

(a) How much of the substance is left after $t$ days?
(b) After how many days will 10 grams of the substance be remaining?
(c) You want to perform an experiment with the substance, but you cannot perform it until 70% of the substance has decayed. How long do you have to wait?

(9) For each of the following difference equations: find $x_2$, then find the equilibria.

(a) $x_{n+1} = x_n^2, x_0 = 2$
(b) $x_{n+1} = \frac{1}{x_n}, x_0 = 2$
(c) $x_{n+1} = \sqrt{x_n + 2}, x_0 = 0$

(10) John takes out a loan from a bank at 2% interest per month. He will be able to pay back 20 dollars per month. Let the initial loan amount be $L$, and let $x_n$ be the amount of money John still owes after $n$ months (so $x_0 = L$).

(a) Find $x_2$ in terms of $L$.
(b) Find a difference equation relating $x_{n+1}$ and $x_n$.
(c) Find the equilibria of this system.
(d) Interpret this result (i.e. what will happen if the initial loan amount is bigger than or less than this equilibrium)? (Hint: use cobwebbing)
Answers:

(1) (a) No
(b) Yes
(c) No
(d) No
(e) Yes

(2) (a) log-log
(b) Draw the points (√W, √L)
(c) a = 2.3 and b = 2

(3) Check with graphing calculator or WolframAlpha

(4) \( f(x) = 3 \cdot 2^x \)

(5) (a) After 1 year: 600 dollars, after 2 years: 720 dollars, after \( t \) years: \( 500(1.2)^t \) dollars.
(b) After 1 year: \( 500(1.1)^2 \) dollars, after 2 years: \( 500(1.1)^4 \) dollars, after \( t \) years: \( 500(1.1)^{2t} \) dollars.
(c) \( 500 \left( 1 + \frac{2}{n} \right)^n \) dollars.
(d) \( 500e^{-2} \) dollars.

(6) (a) −2
(b) 3

(7) (a) ln(2) − 2
(b) \( e/2 \)
(c) 1/4

(8) (a) \( 100(1/2)^{t/2} \)
(b) \( 2\ln(1/10)/\ln(1/2) \) days (simplified)
(c) \( 2\ln(3/10)/\ln(1/2) \) days.

(9) (a) \( x_2 = 16 \), equilibria 0 and 1
(b) \( x_2 = 2 \), equilibria at \( \pm 1 \).
(c) \( x_2 = \sqrt{2 + \sqrt{2}} \), equilibrium at 2

(10) (a) \( x_2 = 1.02(1.02L - 20) - 20 \)
(b) \( x_{n+1} = 1.02x_n - 20 \)
(c) equilibrium at 1000.
(d) If the initial \( L \) is less than 1000, then John will be able to pay off his loans. Otherwise, he will not be able to, and he will get more and more in debt.