

Math 32A - Midterm 1

1. (10 points) Suppose that a swan (not a duck!!) is swimming in a circle described by the path

$$c(t) = (\cos t, \sin t), \quad t \in \mathbb{R}^+$$

and that the water temperature is given by

$$T(x, y) = x^2 e^y \quad x, y \in \mathbb{R}$$

(a) Find $DT(x, y)$ and explain what this represents.

(b) Find $D(T \circ c)(t)$ and explain what this represents.

2. (10 points) Determine whether the following statements are true or false. If a statement is true, indicate how you could show it. If a statement is false, provide a counterexample.

(a) If $f(x, y, z) = x^4 + xy + z^3$, then $\nabla f(1, 0, 1)$ is perpendicular to the surface $f=2$ at the point $(1, 0, 1)$.

This statement is (circle one): true false

Explanation or counterexample:

(b) If $f(x, y) = \ln y$, then $\nabla f(x, y) = \frac{1}{y}$

This statement is (circle one): true false

Explanation or counterexample:

3. (10 points) Find the absolute maxima and minima of the function

$$f(x, y) = 5x^2 - 2y^2 + 2$$

on the disk $x^2 + y^2 \leq 1$

4. (10 points) Pictured are a contour map of f and a (dashed) curve with equation $g(x, y) = 8$. Estimate the maximum and minimum values of f subject to the constraint that $g(x, y) = 8$. Explain your choices.