HOMEWORK 1

Ch 5.1

2. (a) For integers $m < n$ number of integers between m and n (inclusive) is $n - m + 1$. Thus the answers is 40-0+1=41.
(b)$\#\{n : n \text{ is even integer}, 0 \leq n \leq 40\} = \#\{2k : k \text{ is integer }, 0 \leq k \leq 20\} = \#\{k : k \text{ is integer }, 0 \leq k \leq 20\}$=20-0+1=21.
(c) $\#\{(0,5),(1,6),(2,7),...,(34,39),(35,40)\} = \#\{0,1,2,...,34,35\} = 35 - 0 + 1 = 36$.

6. There are n married couples, hence n men and n women. We are asked to find the number of ways to pick a man and a woman who are not married. First choose a man (we can do it in n different ways) then choose any woman except his wife (we can do it in $n - 1$ ways). Thus the result is $nx(n-1) = n^2 - n$. OR Now consider the complement. First find the number of ways to pick a married couple and then subtract it from the # of all outcomes. Clearly there are n such pairs. # of all possible otucomes=# of all pairs (one man one woman)=nx$n = n^2$. Finally we get $n^2 - n$.

9. (a) There are 4 aces in a deck, so for the first card there are 4 ways. There are 51 cards left. Since the second card should not be a Queen there are 51-4=47 ways to pick the second card. Thus totally there are 4x47=158 ways.
(b) First case: the first card is Queen of spades. Obviously there are 1 way to choose the first card. Note that we have picked a Queen, thus there are 48 options for the second card. So totally we get 1x48=48 ways.
Second case: the first card is a spade but not a Queen. Except the Queen of spades there are 12 spade cards in a deck. For the second card we have 51-1-4=47 choices since we have chosen a card that is not a Queen. So there are 12x47 ways.
Therefore the answer is 48+12x47.

18. (a) There are two types of such license plates, LLLNNN and NNNLLL where L and N denote letter and number, respectively. Since repetition is allowed for each case there are 26^3x10^3 sequences. Thus the answer is 2x26^3x10^3.

29. Case 1: Jack picks an apple. He can do it in 15 different ways. Then Jill picks one apple from the remaining 14 apples and one pear from 10 pears.# of possible outcomes for Jill is 14x10. Hence for this case # of total outcomes is 15x14x10.
Case 2: Jack picks a pear. He has 10 options. Then Jill takes one of 15 different apples and one of 9 different pears. Therefor in this case # of possible outcomes is 10x15x9. So the answer is 15x14x10+10x15x9=10x15(14+9)=10x15x24.

Ch 5.2

2. # of arrangements of k different objects chosen from n different objects is $P(n,k)$. In this problem $n = 26, k = 8$. Then the answers is $P(26,8)$. OR for the first letter there are 26 choices, for the second letter there are 25 options since we can not use the first letter. Similarly for the third letter there are 24 options, ..., for the eighth letter there are 19 options. Thus the answer is 26x25x....x19.

6. # of ways to pick a subset of k elements from a set of n elements is $C(n,k)$. here $n = 26, k = 6$, so the answer is $C(26,6)$.

9. The answer is

$$\frac{\text{\# of arrangements of 7 Hs and 2 Ts } + \text{\# of arr. of 8 Hs and 1 T } + \text{\# of arr. of 9 Hs}}{\text{\# of nine letter sequences of H and T}}$$

$= \frac{9!/7!2!+9!/8!1!+9!/9!}{2^9} = \frac{36+9+1}{512} = \frac{46}{512} = \frac{23}{256}$

10. For the first position there are 2 options, a and e, then for the sixth place there is only one option. And b,c,f,g are ordered in second, third, fourth and fifth places in 4! ways. thus # of arrangements satisfying conditions is 2x4!. # of all arrangements is 6!. Then the answer is $\frac{2\text{x}4!}{6!} = \frac{1}{15}$.

24. The probability that a random 9-digit SSN has at least one repeated digit= 1- the probability that a random 9-digit SSN has no repeated digit
=1- $\frac{\text{\# of 9−digit SSNs with no repeated digit}}{\text{\# of all 9−digit SSNs}} = 1 - \frac{P(10,9)}{10^9} = 1 - \frac{10!}{10^9}$.