Problem 1. Let \( a_1, a_2, \ldots, a_n \) be positive numbers. Prove the general AM-GM inequality:
\[
a_1 + a_2 + \ldots + a_n \geq n \sqrt[n]{a_1 a_2 \ldots a_n}.
\]
When does equality occur?

Problem 2. Let \( a_1, a_2, \ldots, a_n \) and \( b_1, b_2, \ldots, b_n \) be non-zero real numbers. Prove Cauchy-Schwarz’s inequality:
\[
(a_1^2 + a_2^2 + \ldots + a_n^2)(b_1^2 + b_2^2 + \ldots + b_n^2) \geq (a_1 b_1 + a_2 b_2 + \ldots + a_n b_n)^2.
\]
When does equality occur?

Problem 3. Let \( a, b, c \) be positive numbers. Show that we have
\[
\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} \geq \frac{(x + y + z)^2}{a + b + c}
\]
for every \( x, y, z > 0 \).

Problem 4

(a) Show that for every real numbers \( x, y, z \), we have
\[
x^2 + y^2 + z^2 \geq xy + yz + zx.
\]
When does equality occur?

(b) Using part (a) and Problem 3 to prove Nesbitt’s inequality:
\[
\frac{x}{y + z} + \frac{y}{z + x} + \frac{z}{x + y} \geq \frac{3}{2}
\]
for every positive numbers \( x, y, z \).