Problem 1.

(a) We use chain rule where the inner function is $x^3 + 1$ and the outer function is $\cos^2 x$. The derivative of the outer function $\cos^2 x$ is $2 \cos x (-\sin x)$. The derivative of the inner function is $3x^2$. Put these results together, we get

$$\frac{d}{dx} \cos^2(x^3 + 1) = -2 \cos(x^3 + 1) \sin(x^3 + 1) \cdot 3x^2$$

$$= -6x^2 \cos(x^3 + 1) \sin(x^3 + 1).$$

Note that we have the trig identity $\sin(2u) = 2 \cos u \sin u$; the answer can also be written as $-3x^2 \sin(2x^3 + 2)$.

(b) First we use the product rule

$$\frac{dg}{dx} = \ln(x^2 - 2) \frac{d}{dx} x + x \frac{d}{dx} \ln(x^2 - 2).$$

To compute the derivative of $\ln(x^2 - 2)$, we use the chain rule as part (a) with outer function $\ln x$ and inner function $x^2 - 2$. The derivative of $\ln(x^2 - 2)$ is computed as $\frac{1}{x^2 - 2} \cdot (2x)$. Therefore we have

$$\frac{dg}{dx} = \ln(x^2 - 2) + x \cdot \frac{2x}{x^2 - 2}$$

$$= \ln(x^2 - 2) + \frac{2x^2}{x^2 - 2}.$$

(c) Using the quotient rule

$$\frac{dh}{dx} = x \frac{d}{dx} e^{x^2 + x} - e^{x^2 + x} \frac{d}{dx} x.$$  

By chain rule, we can compute $\frac{d}{dx} e^{x^2 + x} = (2x + 1)e^{x^2 + x}$. Therefore the answer is $\frac{e^{x^2 + x}(2x^2 + x - 1)}{x^2}$.
**Problem 2.** Compute the following integrals

(a) Note that that \( x^n \) has an antiderivative \( \frac{x^{n+1}}{n+1} \) for each integer \( n \neq -1 \). Therefore the integrand has an antiderivative \( x^3 + \frac{x^2}{2} + x \). The integral is evaluated to be

\[
\int_0^1 \left( \frac{1^3}{3} + \frac{1^2}{2} + 1 \right) - \left( \frac{0^3}{3} + \frac{0^2}{2} + 1 \right) = \frac{11}{6}.
\]

(b) Using integration by parts with \( u = x \) and \( dv = e^x \, dx \). So \( du = dx \) and \( v = e^x \). Therefore we have

\[
\int xe^x \, dx = xe^x - \int e^x \, dx,
\]

which in turn equals to \( xe^x - e^x + C \). Note that we need to add a constant term \( C \) because the integral is indefinite (no upper and lower limits).

(c) Using change of variable \( u = \ln x \). Then \( x = e^u \) and \( dx = e^u \, du \). The integral becomes

\[
\int \ln x \, dx = \int ue^u \, du = ue^u - e^u + C
\]

by part (b). Substitute \( u \) with \( \ln x \), the integral equals to \( (\ln x)e^{\ln x} - e^{\ln x} + C \), which equals to \( x \ln x - x + C \).

**Problem 3.** Compute the following limits

(a) The limit is

\[
\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x \to 1} (x+1) = 2.
\]

(b) Note that \(-n+1\) goes to \(-\infty\) as \( n \to +\infty \), the limit has the form \( e^{-\infty} \), which is 0.

(c) The limit has the form \( \frac{\infty}{\infty} \), so we can use L’Hopital rule

\[
\lim_{x \to +\infty} \frac{\ln x}{x} = \lim_{x \to +\infty} \frac{1}{x} = \lim_{x \to +\infty} (1/x) = 0.
\]

(d) \( \lim_{x \to 0} (x \cos x) = 0 \cdot 1 = 0 \).

**Problem 4.** Draw the graphs of the following curves

(a) This is the parabola which is concave up with vertex \((0,1)\).

(b) This is the circle whose center is \((1,0)\) and radius \( r = 2 \).