Problem 1 One of the diagonals is $\vec{v} + \vec{w}$ and the other is $\vec{v} - \vec{w}$. Since they are orthogonal, the dot product $(\vec{v} + \vec{w}) \cdot (\vec{v} - \vec{w})$ is 0. Distributing the product gives us $\|\vec{v}\|^2 - \|\vec{w}\|^2 = 0$. Therefore it must be true that $\|\vec{v}\| = \|\vec{w}\|$.  

Problem 2 Square both sides of the identity gives us $\|\vec{v}\|^2 - 2\vec{v} \cdot \vec{w} + \|\vec{w}\|^2 = \|\vec{v}\|^2 + 2\vec{v} \cdot \vec{w} + \|\vec{w}\|^2$. It follows that $\vec{v} \cdot \vec{w} = 0$. So $\vec{v}$ and $\vec{w}$ are orthogonal.  

Problem 3 (a) $\vec{r}(t) = \langle 1, 2, -1 \rangle + t\langle -1, 0, 2 \rangle$.  
(b) Normal vector $\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \langle 2, -2, -1 \rangle \times \langle -1, -6, 0 \rangle = \langle -6, 1, -14 \rangle$. So the equation of the plane is $-6(x - 0) + 1(y - 1) - 14(z - 1) = 0$ which can be simplified to $-6x + y - 14z = -13$.  
(c) We know that $\vec{j}$ is perpendicular to $xz$-plane, so we can choose direction vector $\vec{v} = \vec{j} = \langle 0, 1, 0 \rangle$. So the equation of the line is $\vec{r}(t) = \langle 2, 4, 1 \rangle + t\langle 0, 1, 0 \rangle$.  
(d) The parallel planes have the same normal vectors. So we can choose $\vec{n} = \langle 1, -1, -1 \rangle$.  

Problem 4 Find the point of intersection of  
(a) We need to find $t$ and $s$ such that $\langle 1, 0, 0 \rangle + t\langle -3, 1, 0 \rangle = \langle 0, 1, 1 \rangle + s\langle 2, 0, 1 \rangle$.  

This simplifies to \((1 - 3t, t, 0) = (2s, 1, 1 + s)\). It follows that \(1 - 3t = 2s\), \(t = 1\) and \(s = -1\). The solution \(t = 1\) and \(s = -1\) is consistent with \(1 - 3t = 2s\). So the two lines intersect at \((1 - 3, 1, 0) = (-2, 1, 0)\).

(b) Rewrite the equation for the line as \(x = -1 + 3t, y = -4 + t, z = 2 + 3t\). Plug these into the equation of the plane

\((-1 + 3t) - 2(-4 + t) - (2 + 3t) = 1\)

we can solve for \(t = 2\). So \(x = -1 + 3t = 5, y = -4 + t = -2\) and \(z = 2 + 3t = 8\) are the coordinates of the intersection.

**Problem 5**

(a) Use the volume of parallelepiped method. Check if \(\overrightarrow{PQ} \cdot (\overrightarrow{PS} \times \overrightarrow{PR}) = 0\).

(b) The area is \(\frac{1}{2}\|\overrightarrow{PQ} \times \overrightarrow{PR}\|\).

**Problem 6** They all equal to the volume of the parallelepiped spanned by \(\vec{u}, \vec{v}, \text{ and } \vec{w}\).

**Problem 7** Drop \(PQ\) perpendicular to the plane (\(Q\) is on the plane), then \(PQ\) is the shortest distance. Note that \(\overrightarrow{PQ}\) is normal the plane, so it is parallel to \(\overrightarrow{n} = (1, 2, 1)\). Thus \(\overrightarrow{PQ} = \lambda\langle 1, 2, 1 \rangle\) for some scalar \(\lambda\). Since \(P\) is given, one can use this to solve for \(x_Q, y_Q, z_Q\) in terms of \(\lambda\). Since \(Q\) is on the plane, plug these coordinates of \(Q\) (in terms of \(\lambda\)) into the equation of the plane to find \(\lambda\).

**Problem 8**

(a) It is the circle at height \(z = 1\)

(b) \(\vec{r}(0) = \langle 1, 0, 1 \rangle, \ \vec{v}(t) = \langle -\sin t, \cos t, 0 \rangle\). So \(\vec{v}(1) = \langle -\sin 1, \cos 1, 0 \rangle\).

(c) \(L(t) = \vec{r}(2) + t\vec{v}(2) = \langle \cos 2, \sin 2, 1 \rangle + t\langle -\sin 2, \cos 2, 0 \rangle\).

(d) One can compute that \(\|\vec{r}(t)\| = \sqrt{2}\) for all \(t \geq 0\). This shows the orthogonality of \(\vec{r}(t)\) and \(\vec{v}(t)\). (See textbook page 717 Example 7).

**Problem 9** Their normal vectors are \(\vec{n}_1 = \langle 1, 2, -1 \rangle\) and \(\vec{n}_2 = \langle -2, -4, 2 \rangle\) respectively. Since \(\vec{n}_2 = -2\vec{n}_1\), they are parallel. Thus the two planes are parallel. To find their distance, pick any point \(P\) on the first plane. Drop \(PQ\) perpendicular on the second plane and proceed like Problem 4.

**Problem 10** The midpoint \(M\) has the coordinates \(x_M = (0 + 1)/2 = 1/2, y_M = (1 + 2)/2 = 3/2, z_M = (0 + 1)/2 = 1/2\).
\[ z_M = (\frac{-2+4}{2})/2 = 1. \] The line is normal to the plane \( 2x - y - z = 1 \), so it has a direction vector \( \vec{v} = (2, -1, -1) \). Thus the equation of the line is \( \vec{r}(t) = \langle \frac{1}{2}, \frac{3}{2}, 1 \rangle + t\langle 2, -1, -1 \rangle \).

**Problem 11**  True or false:

(a) True.

(b) False.

(c) True.