Math 32A Midterm 1 Practice Problems

Sections 13.1-13.6 and 14.1, 14.2

October 19, 2018

Study tips

• Make sure you go over (most) examples provided in the textbook.
• Know/memorize the definitions and terms highlighted in the textbook (e.g. triple product, equivalent vectors, parallelepiped, etc.)
• Memorize all the theorems and formula covered in lectures.
• Understand the differences between dot product and cross product.
• After finding the intersection of 2 planes or 2 lines, plug the answer back to the original equations of the lines or planes to double check.
• Do all the homework problems.

Problem 1 Suppose the parallelogram spanned by two non-zero vectors \( \vec{v} \) and \( \vec{w} \) has its diagonals orthogonal to each other. Show that we have \( \|\vec{v}\| = \|\vec{w}\| \).

Problem 2 Suppose two non-zero vectors \( \vec{v} \) and \( \vec{w} \) satisfy \( \|\vec{v} - \vec{w}\| = \|\vec{v} + \vec{w}\| \). Show that \( \vec{v} \) and \( \vec{w} \) are orthogonal.

Problem 3 Write a parametrization of

(a) The line passes through \( S = (1, 2, -1) \) and parallel to \( (-1, 0, 2) \).
(b) The plane spanned by \( P = (0, 1, 1) \), \( Q = (2, -1, 0) \) and \( R = (-1, -5, 1) \).
(c) The line perpendicular to the zx-plane and passes through the point \((2, 4, 1)\).
(d) The plane contains \((0, 1, 1)\) and is parallel to the plane \(x - y - z = 1\).

Problem 4 Find the point of intersection of
(a) The lines \( \mathbf{r}_1(t) = (1,0,0) + t(-3,1,0) \) and \( \mathbf{r}_2(t) = (0,1,1) + t(2,0,1) \).
(b) The line \( \mathbf{r}(t) = (-1,-4,2) + t(3,1,3) \) and the plane \( x - 2y - z = 1 \).

**Problem 5** Let \( P = (-1,0,2) \), \( Q = (2,1,0) \), \( R = (-1,1,2) \) and \( S = (2,0,1) \).
(a) Do \( P, Q, R, S \) lie on the same plane? Explain.
(b) Find the area of the triangle \( PQR \).

**Problem 6** Let \( \mathbf{u}, \mathbf{v}, \mathbf{w} \) be three vectors in \( \mathbb{R}^3 \). Show that

\[
|\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = |\mathbf{v} \cdot (\mathbf{w} \times \mathbf{u})| = |\mathbf{w} \cdot (\mathbf{u} \times \mathbf{v})|.
\]

*Hint: Interpret each term geometrically.*

**Problem 7** Find the shortest distance from \( P = (0,1,-5) \) to the plane \( x + 2y + z = 1 \).

**Problem 8** A particle is moving in \( \mathbb{R}^3 \) with its position at time \( t \) is given by \( \mathbf{r}(t) = (\cos t, \sin t, 1) \) where \( t \geq 0 \).
(a) Sketch the orbit of the particle.
(b) What is the initial position of the particle? What is its velocity vector at \( t = 1 \)?
(c) Find the tangent line at \( \mathbf{r}(2) \).
(d) Show that the velocity vector \( \mathbf{v}(t) \) is always tangent to the orbit (in other words, \( \mathbf{r}(t) \) is orthogonal to \( \mathbf{v}(t) \)).

**Problem 9** Show that the two planes \( x + 2y - z = 2 \) and \( -2x - 4y + 2z = 3 \) are parallel and find the distance between them.

**Problem 10** Given two points \( A = (0,1,-2) \) and \( B(1,2,4) \). Find an equation of the line passes through the midpoint \( M \) of the segment \( AB \) and is normal to the plane \( 2x - y - z = 1 \).

**Problem 11** True or false:
(a) Two non-parallel lines in 2D intersect each other.
(b) Two non-parallel lines in 3D intersect each other.
(c) Two planes in 3D are either parallel or they intersect.