Math 21b: Introduction

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TF Ryan Reich (ryanr@math.harvard.edu) SC 428a
CA Chang Xu (changxu@fas.harvard.edu)
Course head Oliver Knill (www.courses.fas.harvard.edu/∼math21b/)
Office hours TBD
Problem session TBD
Textbook *Linear Algebra with Applications* by Otto Bretscher, ed. 3E
Homework Read §1.1; do problems 8,14,20,24,46; optional (but recommended): 26,36

Hi everyone; welcome to Math 21b, Linear Algebra.

I'm Ryan Reich, a PhD student here at Harvard, and you are all very lucky: it's not every day that one gets to learn something entirely new, yet in this class I think you'll find that this does happen almsot every day.

You probably spent most of your time in calculus doing strange things to individual functions — differentiating and integrating them, sketching their graphs, extracting optimum values — but one thing you almost never do there is to find solutions to equations like

$$f(x) = 0, \text{ e.g. } xe^x - 2 = 0.$$

Calculus doesn't give you exact answers to questions like this.

In linear algebra, we will do nothing but discuss solutions to a very specific, relatively simple kind of equation, like

$$\begin{cases} x + 2y + 3z = 39 \\ x + 3y + 2z = 34 \\ 3x + 2y + z = 26 \end{cases}$$

Many of you have probably seen equations (or *systems of equations*) like this before.

Who here has?

Pause to observe hands

Well, you will have a chance a little later to show your stuff.

(If there are others) And if you haven't, don't worry: on Wednesday we will learn the first of the following three general goals for the semester:

- mechanical procedures for finding solutions to these equations;
- a whole framework for describing and analyzing the equations themselves as well as their solutions;
- what structure is given to the "vector spaces" where these solutions live.

We will occasionally descend as though from Olympus to use our powers in concrete situations that are useful to other mathematicians and scientists.

We will even be able to answer a few questions that are not obviously even a part of linear algebra!

Before I go on, let's take a survey of your interests; this is also a good way to learn your names, though I will have to apologize in advance for not being able to remember them just yet (I'll have them by the end of the week, though).

Go around the room and ask their names and what, if any, other scientific courses they are taking or might take. Comment on the applications of linear algebra:

Experimental sciences in general *Least squares!*

Biology *Dynamical systems*

Statistics *Regression analysis: correlation of factors, least squares fitting*

Economics *Statistics, perhaps dynamical systems*

Here is a brief, friendly outline of the semester:

1. How to solve any system of linear equations.
2. Linear transformations and vector spaces, or: what were we really doing last week?
3. Orthogonality: using geometry in the service of algebra.
   * Least squares fitting: mathematical candy!
4. Advanced properties of linear transformations (this one has no cute name).
5. Applications.
   a. Linear ordinary differential equations.
      * *non*linear ODEs.
   b. Fourier analysis: music to our ears.
      * Partial differential equations.

Although there's no reason you should have understood all of these terms, is anyone insatiably curious what something means?

pause for questions

As you can see, there's a lot of stuff to learn, and you'll learn it better if you learn it more than once.

Therefore I *strongly encourage* you, though I do not require it, that you read each section of the textbook the night before we cover it in class.

I will do my best neither to repeat the book nor to assume it, but to complement it in a way that you will learn more from both of us together than from either alone.

Before giving you a motivational problem, let me introduce our CA, Chang Xu. Chang, I'll give the stage to you for a few minutes.

Leave 5 min. to introduce himself, talk about problem sessions, pass out forms

I'll also be deciding my office hours from your time preferences, so please be thorough and as honest as you can in this shopping period.

Let me also spend just a moment on the course structure.

We will have:

daily homework due at the beginning of class	20%	
two hour-long exams (4 Mar., 8 Apr @ 7PM)	20%	(each)
one Mathematica "lab"	5%	
a long, grueling final	35%	

Any questions about this unpleasant topic?

pause for questions. Concerning Mathematica: it can do any computation, but it doesn't know what they mean. You have to learn what questions to ask it and what the answers mean. In order to help you understand the theory, you have to get used to the computations: it connects two completely different areas of the brain and the redundancy helps you learn.

We'll get all your work back to you as soon as possible so you always know where you stand; hopefully, by the next class meeting in all cases except the final exam, which you will never see again.

So that *I* always know where you stand, and because sometimes math only makes sense once you hear it twice, I'll have office hours every week and I *strongly encourage* you to visit, even if you think you only have stupid questions.

Plus, I'll have chocolates for you.

Now, let me give you five minutes to hack away at that linear system I put up earlier — no need to worry if you don't know what to do, or if you don't have fancy tricks; this is just warm-up for Wednesday.

5 minutes for them to work

spend the rest of the time going over their solutions