## HW #7

1.(*) Let $G$ be a finite $p$-group. Show if $p^n \mid |G|$ then $G$ has a normal subgroup of order $p^n$.

2. Let $G$ be a finite group with subgroups $H$ and $K$. Recall that $HK := \{hk \mid h \in H, \, k \in K\}$. Show that $|HK| = |H||K|/|H \cap K|$.

3.(*) Let $G$ be a finite group with normal subgroups $H$ and $K$ of relatively prime order. Show that the group $HK$ is abelian if $H$ and $K$ are abelian and cyclic if $H$ and $K$ are cyclic.

4. Suppose a group $G$ has $r$ distinct subgroups of prime order $p$. Show that $G$ contains at least $r(p-1)$ elements of order $p$. In particular, show that any group of order 56 has a proper normal subgroup.

5. Show that every group of non-prime order less than 60 has a proper normal subgroup.