

HW #6

- 1.(*) Let G be a group and S a non-empty set. Show
 - a. If $\star : G \times S \rightarrow S$ is a G -action then $\phi : G \rightarrow \Sigma(S)$ by $\phi(x)(s) = x \star s$ (i.e., if we let $\phi_x = \phi(x)$ then $\phi_x(s) = x \star s$) is a group homomorphism (called the **permutation representation**).
 - b. If $\phi : G \rightarrow \Sigma(S)$ is a group homomorphism then $\star : G \times S \rightarrow S$ by $x \star s = \phi_x(s)$ where $\phi_x = \phi(x)$ is a G -action.
2. Let $H \subset G$. Define an equivalence relation on G by $a \equiv b \pmod{H}$ if $ab^{-1} \in H$. [This gives rise to right cosets.] Find the group A and the set S and a left A -action on S so that the equivalence classes of this action are the right cosets of H in G .
3. Read Sections 1.3 and 3.5 of the book. Compute all the conjugacy classes and isotropy subgroups in A_4 .
4. Let G be a group of order p^n , p a prime. Suppose the center of G has order at least p^{n-1} . Then G is abelian.
- 5.(*) Let G be a finite group. Suppose that p is the smallest prime dividing the order of G . Suppose also that there exists a subgroup H of G of index p . Then $H \triangleleft G$.
6. Let G be a group with a subgroup H of finite index. Then H contains a subgroup N that is normal and of finite index in G