Math 115AH Homework #9

Problem 1. Let V be a finite dimensional inner product space over F and $T : V \to V$ linear. Show that $\operatorname{im}(T^*) = \ker(T)^\perp$.

Problem 2. Let V be a finite dimensional complex inner product space and $T : V \to V$ a linear operator. Show that T is hermitian if and only if $\langle Tv, v\rangle$ is real for all $v \in V$.

Problem 3. Let $T : V \to V$ be linear where V is a complex finite dimensional inner product space. Show that T is normal if and only if $T = T_1 + \sqrt{-1}\, T_2$ for some commuting hermitian operators $T_1, T_2 : V \to V$.

Problem 4. Let T be an isometry of $\mathbf{R}^3$. Suppose that $\det T = 1$, i.e., the determinant of any matrix representation of T is one. Show that T is a rotation about some axis.