Homework #6

Read Hoffman and Kunz Section 3.7 (and finish 3.5-3.6). We shall continue going over material from these sections in the 4:00 session Wednesday.

Problem 1. Let V be a finite dimensional real vector space of odd degree. Show that any linear operator on V has an eigenvalue. Is this true if V is even dimensional? Prove of disprove.

Problem 2. Let $T : V \to V$ and $S : V \to V$ be a linear operators such that $T \circ S = S \circ T$. Let $E_T(\lambda)$ be an eigenspace of T. Show that $S(E_T(\lambda)) \subset E_T(\lambda)$.

Problem 3. We call a matrix $A \in \mathbf{M}_n(F)$ **nilpotent** if $A^m = 0$ for some positive integer m. Show that if A is nilpotent, then the only eigenvalue of A is zero.

Problem 4. Let $A \in \mathbf{M}_n(F)$. Show that A and A^t have the same eigenvalues.

Problem 5. A matrix $A \in \mathbf{M}_n(F)$ is called **upper triangular** if

$$A_{ij} = 0 \quad \text{for all} \quad 1 \leq j < i \leq n.$$

Suppose that $F = \mathbf{C}$, so every non-constant complex polynomial has a root (by the **Fundamental Theorem of Algebra** which we assume). Show that any matrix in $\mathbf{M}_n(\mathbf{C})$ is similar to an upper triangular matrix. In particular, if $T : V \to V$ is a linear operator with V a finite dimensional complex vector space, then there exists an ordered basis $\mathcal{B}$ for V such that $[T]_{\mathcal{B}}$ is upper triangular.

Problem 6. Hoffman, Kunze p. 190 Problem 6.2/9

Problem 7. Hoffman, Kunze p. 190 Problem 6.2/13

Problem 8. Hoffman, Kunze p. 106 Problem 3.5/11

Problem 9. Hoffman, Kunze p. 115 Problem 3.7/3

Problem 10. Hoffman, Kunze p. 116 Problem 3.7/7