Mathematics 110AH

HW #6

1.(*) Let G be a group and S a non-empty set. Show

a. If $\star : G \times S \to S$ is a G-action then $\phi : G \to \Sigma(S)$ by $\phi(x)(s) = x \star s$ (i.e., if we let $\phi_x = \phi(x)$ then $\phi_x(s) = x \star s$) is a group homomorphism (called the **permutation representation**).

b. If $\phi : G \to \Sigma(S)$ is a group homomorphism then $\star : G \times S \to S$ by $x \star s = \phi_x(s)$ where $\phi_x = \phi(x)$ is a G-action.

2. Let $H \subset G$ be a subgroup. Define an equivalence relation on G by $a \equiv b \pmod{H}$ if $ab^{-1} \in H$. [This gives rise to right cosets.] Find the group A and the set S and a left A-action on S so that the equivalence classes of this action are the right cosets of H in G.

3. Compute all the conjugacy classes and isotropy subgroups in A_4. (Read the Notes Section 24.1 through 24.9.)

4. Let G be a group of order p^n, p a prime. Suppose the center of G has order at least p^{n-1}. Prove that G is abelian.

5.(*) Let G be a finite group. Suppose that p is the smallest prime dividing the order of G. Suppose also that there exists a subgroup H of G of index p. Prove that $H \triangleleft G$.

6. Let G be a group with a subgroup H of finite index. Prove that H contains a subgroup N that is normal and of finite index in G