SOLVABLE LATTICE MODELS, MATH 216A

RAPHAËL ROUQUIER

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MWF 10-10:50am, MS 5138

This course is an introduction to the theory of solvable lattice models. Lattice models provide a mathematical framework for studying statistical mechanics systems. Some of those systems are exactly solvable, meaning their physical quantities can be computed explicitly. These are two-dimensional systems that satisfy an invariance condition governed by the Yang-Baxter equation. A solution of that equation leads to an algebraic structure, a quantum group. The representation theory of that quantum group determines the transfer matrix, which in turn governs the partition function of the system. Baxter's solution of the 6-vertex model follows from a property of representations of affine quantum groups, the existence of Baxter polynomials, as shown by Frenkel and Hernandez.

Topics to be discussed:

- Yang-Baxter equation
- 2d Ising, ice and 8-vertex models
- Integrable systems
- Lax pairs, (quantum) inverse scattering
- XXX, XXY and XYZ Heisenberg spin chains
- Affine quantum groups, representations and transfer matrices.

Prerequisites: basic representation theory and Lie theory.

Office Hours: by appointment.

Grading: Grading will be based on class attendance and participation.

References:
Rodney J. Baxter, *Exactly Solved Models in Statistical Mechanics.*
Christian Kassel, *Quantum Groups.*

George Lusztig, *Introduction to Quantum Groups*.
Jean-Pierre Serre, *Lie Algebras and Lie Groups*.
Jean-Pierre Serre, *Complex Semi-Simple Lie Algebras*.