ERRATA

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1. Isométries parfaites dans les blocs à défaut abélien des groupes symétriques et sporadiques

p.659, Corollary 2.10: Replace "x est de e-type σ, comme élément de $\mathfrak{S}_{we}$" by "x est de la forme $x = (f;\sigma)$".

p.665, §3.3.1, l.4: One should read "$C_G(x) = C_H(x) = 3 \times \mathfrak{S}_3$".

p.668, §3.3.4, l.4: One should read "$\tilde{H} = N_{\tilde{G}}(P) = H.2$ (non-trivial extension)".

2. The derived category of blocks with cyclic defect groups

- p.209, proof of Lemma 2.13. "$\phi \in \mathrm{End}_{\mathcal{O}}(M)$" should be replaced by "$\phi \in \mathrm{End}_{\mathcal{O}}(M)^{\times}$".
- p.220. In the Brauer tree, "$P_{\frac{p-\epsilon}{4}}$" should be "$P_{\frac{p+\epsilon-2}{4}}$".
- p.220, Read "Restricting a surjective map $\bigoplus_{0\le\lambda\le\frac{p-3}{2}}\ldots$".

3. Centers and simple modules for Iwahori-Hecke algebras

The proof of the first implication in Theorem 3.3 (i) (the "easier" part) is incomplete, and the following should be added (as discussed with M. Geck).

The setting is that of Theorem 3.3. We assume the map $1_k \otimes d_{\mathcal{O}/\mathfrak{p}} : k \otimes \bar{R}_0(k_{\mathfrak{p}}H) \to k \otimes \bar{R}_0(kH)$ is an isomorphism.

By duality, the map $1_k \otimes e_{\mathcal{O}/\mathfrak{p}} : k \otimes \bar{K}_0(kH) \to k \otimes \bar{K}_0(k_{\mathfrak{p}}H)$ is an isomorphism. It follows that $1_{\mathcal{O}/\mathfrak{p}} \otimes e_{\mathcal{O}/\mathfrak{p}}$ is an isomorphism as well.

Let $\tilde{k}$ be a finite separable extension of k neutralizing for kH and A a discrete valuation ring, unramified extension of $\mathcal{O}/\mathfrak{p}$, with residue field $\tilde{k}$. Let $\hat{A}$ be its completion. Then, we have an isomorphism $t_{\hat{A}}^{K_0} : K_0(\hat{A}H) \to K_0(\tilde{k}H) = \bar{K}_0(kH)$. So, we deduce that $1_{\hat{A}} \otimes t_F^{K_0} : \hat{A} \otimes K_0(\hat{A}H) \to \hat{A} \otimes K_0(FH)$ is an isomorphism, where F is the field of fractions of $\hat{A}$. As a consequence, the restriction of $(\cdot,\cdot)_F$ to $K_0(FH) \times \mathcal{F}(\hat{A}H)$ takes values in $\hat{A}$. Since $K_0(FH) = \bar{K}_0(k_{\mathfrak{p}}H)$, it follows that the restriction of $(\cdot,\cdot)_{k_{\mathfrak{p}}}$ to $\bar{K}_0(k_{\mathfrak{p}}H) \times \mathcal{F}((\mathcal{O}/\mathfrak{p})H)$ takes values in $\hat{A} \cap k_{\mathfrak{p}} = \mathcal{O}/\mathfrak{p}$.

The first part of the proof of Theorem 3.3 shows then that the restriction of the bilinear form $(\cdot,\cdot)_{\hat{\tilde{\mathcal{O}}}_{\mathfrak{p}}}$ to $K_0(\hat{\tilde{\mathcal{O}}}_{\mathfrak{p}}) \times \mathcal{F}(H)$ has values in $\mathcal{O}$.

Note that in the proof of Theorem 3.3, we should have written $\mathcal{O}/\mathfrak{p} \otimes \mathrm{ch}\,\bar{R}_0(k_{\mathfrak{p}}H)$ instead of $\mathrm{ch}\,\bar{R}_0(k_{\mathfrak{p}}H)$ (this occurs twice).

Date: May 24, 2021.

4. Complex reflection groups, braid groups, Hecke algebras

Fact 1.7(2) is not correct, as pointed out by Sinead Wilson in "Stabilisers of eigenvectors in complex reflection groups", §3.7.2. It fails for the infinite family $G(r,p,n)$. In that case, every chain of parabolic subgroups is conjugate in $G(r,1,n)$ (but not necessarily in $G(r,p,n)$) to a chain given by admissible subdiagrams. As explained by G. Chapuy and T. Douvropoulos in "Coxeter factorizations with generalized Jucys-Murphy weights and matrix tree theorems for reflection groups" (arXiv:2012.04519), Lemma 4.2, Fact 1.7(2) can be corrected by replacing "$g \in W$" by "$g \in N_{\mathrm{GL}(V)}(W)$".

5. Block theory via stable and Rickard equivalences

p.119, "5-dimensional" should be "4-dimensional".

6. Complexes de chaînes étales et courbes de Deligne-Lusztig

- §4.1.1 p.502, line -17: Remove "Le morphisme $Y \to Y/G^F$ est étale".
- Lemme 4.1 p.502: Read "Soit L un p'-sous-groupe de $G^F \times (T^F)^{\circ}$". Cf O. Dudas and R. Rouquier, "Coxeter orbits and Brauer trees III", J. AMS 27 (2014), 1117-1145, Lemma 2.1 for a more general statement and a proof.

7. Categorification of $\mathfrak{sl}_2$ and braid groups

- In §8.1.3, Theorem 8.3, one should assume in addition that $\mathcal{A}$ is idempotent complete and that η is a split injection.

I thank Paul Balmer for bringing this to my attention. Cf Paul Blamer, "Descent in triangulated categories", Math. Ann. 353 (2012), no. 1, 109–125 for a study of Barr-Beck Theorem in triangulated categories.

- In §9.1.1, replace "We denote by $\{\alpha_s\}_{s\in S}$... for $s \in S$)" by "Given $s \in S$, we denote by α_s an element of V^* such that $\ker(s - \mathrm{id}) = \ker \alpha_s$".

8. Derived equivalences and $\mathfrak{sl}_2$-categorifications

p.278, Theorem 6.6: The assumption "$\lambda \geq 0$" is not necessary for the statement of the theorem nor for its proof.
p.287, l.8: Replace "$(i-1,i)$" by "$(i,i+1)$".

9. Category $\mathcal{O}$ for rational Cherednik algebras

p.620, l.6: Add the requirement that B_i and $\bar{B}_i$ are finitely generated k-modules for all $i \in \mathbf{Z}$.
p.621, l.4: "locally finite" should be replaced by "locally nilpotent".

p.621, l.7: "$\mathrm{Hom}_k(B,k)$ should be replaced by "$\mathrm{Homgr}_k^\bullet(B,k)$".

p.623, Proof of Corollary 2.8 Let P be a projective object of $\mathcal{O}$ such that every $\Delta(E)$ is a quotient of P. By Proposition 2.2, every object M of $\mathcal{O}$ has an ascending filtration $0 = M_0 \subset M_1 \subset \cdots \subset M$ with $M = \bigcup_i M_i$ and M_i/M_{i-1} is a quotient of $\Delta(E_i)$ for some E_i. By assumption, there are morphisms $f_i : P \to M_i$ that induce surjections $P \to M_i/M_{i-1}$. So, $\sum_i f_i : P^{(\mathbf{Z})} \to M$ is a surjection. It follows that P is a progenerator of $\mathcal{O}$. So, Corollary 2.8 follows from Corollary 2.7.

In the definition in §5.2.5 of the Hecke algebra, one should read $\det(s)^j$ instead of $\det(s)^{-j}$.

10. Coxeter orbits and modular representations

As pointed out by H. Wang, the isomorphism in l.18, p.30 is false. This is used in l.19, p.30 to obtain a disjointness result. A correct proof of that disjointness result is given in Proposition 3.4.3 of

H. Wang, L'espace symmétrique de Drinfeld et correspondance de Langlands locale II, preprint (2014), arXiv:1402.1965.

11. Dimensions of triangulated categories

• Proposition 4.8

The statement of the Proposition 4.8 needs to be changed: one assumes $\mathrm{End}^*(X)$ is coherent (*i.e.*, a submodule of a finitely generated module is finitely generated) and one replaces "locally finitely generated" by "locally finitely presented" in the statement of the Proposition.

I thank Hang Xing Chen for pointing out the following issues.

• Proposition 4.13,

(i): One should read "...for any $r \geq 0$, the system $(H_{ni+r})_{i\geq 1}$...".

(iv) One should read "$H_{2n} \to \operatorname{colim} H_i$"

proof of that Proposition

line 3, one should read "Given $i \geq 2$,...".

Let us justify that the left most vertical sequence of the diagram is exact. Let V_i be the homology at the middle of that sequence. There is a surjective map $s_i : H(J) \to V_i$ such that $s_{i+1} = \mathrm{can} \circ s_i$. Since the canonical map $K_i \to K_{i+1}$ evaluates to 0 on $I'[-1]$, it follows that the induced map $V_i \to V_{i+1}$ is 0. We deduce that $V_i = 0$ for $i \geq 2$.

line 10, one should read "By induction...for any $i \geq n$...".

• Proof of Lemma 5.8.

line 9, read "$i_{2*}i_2^! j_{1*}j_1^* C \to$ ...".

In the proof, one should note that since $i_{2*}i_2^* i_{1*}i_1^* \simeq i_{1*}i_1^* i_{2*}i_2^*$, it follows that $i_{2*}i_2^*$ preserves $\mathcal{I}_1$-local objects, hence $j_{2*}j_2^*$ preserves $\mathcal{I}_1$-local objects.

12. Derived equivalences and finite dimensional algebras

Remark 2.14: the conjecture is false. Take $\mathbf{G}$ of type B_2, $\mathbf{L}$ a maximal torus of type w_0. By [Digne, Michel and Rouquier, "Cohomologie des variétés de Deligne-Lusztig", Theorem 3.4], the graded character of $H^*(X_{\mathbf{U}})$ is $1 + q^3(\sigma + \tau + 2\rho) + q^4\mathrm{St}$, where σ and τ are the unipotent characters associated with the linear characters of B_2 different from $1, \varepsilon$ and ρ is the unipotent character associated with the reflection representation.

13. Perverse equivalences and Broué's conjecture

p.10, l.19: P should be T.

14. Representations of rational Cherednik algebras

Section 1: The categories of finite-dimensional modules for rational Cherednik algebras embedd in, but can be smaller than, the ones for trigonometric Cherednik algebras, contrarily to what is stated. On the other hand, trigonometric and elliptic Cherednik algebras have equivalent categories of finite-dimensional modules, cf M. Varagnolo and E. Vasserot, "Finite-dimensional representations of DAHA and affine Springer fibers: the spherical case", Duke Math. J. **147** (2009), 439–540.

15. q-Schur algebras and complex reflection groups

p.131, Proposition 4.19. The category $\mathcal{C}^{\mathrm{opp}}$ is not a module category in general, it should be replaced by A^{opp}-mod, where $\mathcal{C} = A$-mod.

p.132, Lemma 4.21: The proof is incomplete (in the induction, it is not clear that M/M_0 is projective over k). A complete proof is given in R. Rouquier, P. Shan, M. Varagnolo and E. Vasserot, "Categorifications and cyclotomic rational double affine Hecke algebras", Lemma 2.7.

p.143, §5.2.1: The functor KZ should be modified by multiplying the action of an s-generator of the monodromy around H by $\mathbf{q}_{H,0}$.

I would like to thank Iain Gordon for bringing to my attention a gap in the proof of Theorem 5.5 and for his help in fixing it.

We start in the setting of §5.2.1: $\mathfrak{m}$ is a maximal ideal of $\mathbf{C}[\{\mathbf{h}_u\}]$, k' the completion at $\mathfrak{m}$, k the residue field of k'. We denote by K the field of fractions of k'.

We have a bijection

$$\mathrm{Irr}(W) \xrightarrow{\sim} \mathrm{Irr}(K\mathbf{H}),\ E \mapsto \mathrm{KZ}(\Delta(E)).$$

Let $L = \mathbf{C}(\{\mathbf{q}_u^{1/l}\})$ as in §3.1.2: the algebra $L\mathbf{H}$ is split semi-simple. We have a canonical morphism $L \to K$, $\mathbf{q}_u^{1/l} \mapsto e^{2i\pi h_u/l}$ and an induced bijection $\mathrm{Irr}(L\mathbf{H}) \xrightarrow{\sim} \mathrm{Irr}(K\mathbf{H})$. Composing with the bijection above, we obtain a bijection

$$\rho_{h.} : \mathrm{Irr}(W) \xrightarrow{\sim} \mathrm{Irr}(L\mathbf{H}).$$

Let $\tau \in \mathfrak{t}_{\mathbf{Z}}$. We put $\tilde{h}. = h. + \tau$. We have an automorphism γ_τ of L given by $\mathbf{q}_u^{1/l} \mapsto$.

We have a bijection as above $\rho_{\tilde{h}.} : \mathrm{Irr}(W) \xrightarrow{\sim} \mathrm{Irr}(L\mathbf{H})$ and

$$\rho_{\tilde{h}.} = \gamma_\tau^* \circ \rho_{h.}.$$

This gives us a permutation $\sigma_\tau = \rho_{h.}\rho_{\tilde{h}.}$ of $\mathrm{Irr}(W)$. As noted by Opdam, we have $\mathbf{c}_{\sigma_\tau(\chi)} = \mathbf{c}_\chi$. This is due to the fact that the automorphism γ_τ acts trivially on irreducible representations of rank 1 parabolic subalgebras of $\mathbf{H}$, because those are defined over $\mathbf{C}(\{\mathbf{q}_u\})$.

The statement of Theorem 5.5 is now

Theorem 5.5. Assume $x_{H,j} \neq x_{H,j'}$ for all $H \in \mathcal{A}$ and $j \neq j'$. Let $\tau \in \mathfrak{t}_{\mathbf{Z}}$. Assume and assume the order $\leq$ on $\mathrm{Irr}(W)$ defined by $h.$ and the the one $\leq_\tau$ defined by $h. + \tau$ are related by $\chi \leq_\tau \chi'$ if and only if $\sigma_\tau(\chi) \leq \sigma_\tau(\chi')$.

Then, there is an equivalence $\mathcal{O}(h.) \xrightarrow{\sim} \mathcal{O}(h. + \tau)$ of quasi-hereditary covers of $k\mathbf{H}$.

16. Quiver Hecke algebras and 2-Lie algebras

Proof of Proposition 2.1:
line 3: $R_{w'} = R_w - \{(i, i+1)\}$ should be replaced by $R_{w'} = s_i(R_w) - \{(i, i+1)\}$.
line -3: $R_v = R_w \cup \{(j, j+1)\}$ should be replaced by $R_v = s_j(R_w) \cup \{(j, j+1)\}$.

Proof of Theorem 4.25, line 9: "left inverse to Ψ" should be replaced by "left inverse to Φ".

§3.3.3. p.19, lines -4, -5: E_i should be replaced by F_i.