LIE GROUPS AND LIE ALGEBRAS, MATH 229B

RAPHAËL ROUQUIER

Winter 2018

MWF 9-9:50am, MS 7608

This course will present the classical theory of Lie algebras, complex semi-simple Lie algebras and their finite-dimensional representations. The course will end with a discussion of quantum groups and their construction from spaces of representations of quivers.

The course will start with basic properties of Lie algebras. The next topic will be nilpotent and solvable Lie algebras. This will be followed by a study of the Lie algebra $\mathfrak{sl}_2$ and its representations. The course will move to more discrete structures: root systems, Weyl groups and Dynkin diagrams. This will lead to the structure theory of semi-simple Lie algebras. The next topic will be the study of finite-dimensional representations of semi-simple Lie algebras. The final chapter will be an introduction to quantum groups. We will introduce quantum groups as deformations of enveloping algebras of complex semi-simple Lie algebras. We will bring in quivers and representations over finite fields and show that the associated Hall algebras are quantum groups.

References

W.Fulton and J.Harris, "Representation Theory", Springer Verlag, 1991

O.Schiffmann, "Lectures on Hall algebras", arXiv math/0611617

J.P.Serre, "Lie algebras and Lie groups", Lecture Notes in Mathematics 1500, Springer Verlag, 1992

J.P.Serre, "Complex semi-simple Lie algebras", Springer Verlag, 1987

V.S.Varadarajan, "Lie Groups, Lie Algebras and their Representations", Springer 1984

Office hours: by appointment

Projects

The course assessment will be based on a 20mn presentation of an assigned project. The presentations will be held during the week March 12–16. A list of possible projects will be provided on Monday February 5 and a preferred choice, as well as a second and third choice, will need to be sent to me by email by Wednesday Feburary 7. An abstract of what you plan to do for the presentation will be due on Wednesday February 21. You may also suggest your own topic, which I will need to approve.