

3. Prove that the set of all 4x4 matrices with rational entries is countable.

Let M be the set of all 4 X 4 matrices with rational entries. Let Q be the set of all rationals and R be the cartesian product of 16 copies of Q. Then the mapping of M to R, defined by

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \rightarrow (a_{11}, \dots, a_{44})$$

is one-to-one and onto. So, M is countable.