

7. Give two instances of theorems on continuous functions whose proofs require the use of the Bolzano-Weierstrass Theorem.

Solution: First, the theorem is used in proving that if $f(x)$ is continuous on $[a,b]$ then $f(x)$ is bounded on $[a,b]$. One supposes the theorem is false. Then there is a sequence $\{x_n\}$ from $[a,b]$ such that $|f(x_n)| > n$ for $n = 1, 2, \dots$. The Bolzano-Weierstrass theorem is then applied to $\{x_n\}$, and this leads to a contradiction.

Second, The Bolzano-Weierstrass theorem is used in proving that the function f assumes its max on $[a, b]$. One lets

$$A = \{f(x) : x \text{ in } [a, b]\} \text{ and}$$

$u = \sup(A)$. Then (*fill in the rest*)

•
•
