Review, Problem 5

Let

$$S_n = 1^2 + 2^2 + \quad + n^2$$

prove that

$$S_n = (1+2+ \quad +n)(2n+1)/3 \text{ .}$$

Proof: First, use the fact that

$$(1+2+ \quad +n) = \frac{n(n+1)}{2} \text{ .}$$

Then the problem reduces to proving that

$$S_n = n(n+1)(2n+1)/6 \text{ .}$$

A simple computation shows that this is true for n = 1.

So we go on to the inductive step:

$$\begin{aligned} S_{n+1} &= 1^2 + 2^2 + \quad + n^2 + (n+1)^2 \\ &= n(n+1)(2n+1)/6 + (n+1)^2 \\ &= (n+1)(n(2n+1)/6 + (n+1)) \\ &= (n+1)(2n^2 + n + 6n + 6)/6 \\ &= (n+1)(2n^2 + 7n + 6)/6 \\ &= (n+1)(n+2)(2n+3)/6 \\ &= (n+1)(n+2)(2(n+1)+3)/6 \end{aligned}$$

So, the pattern persists; the inductive step is holds.