

Review, Problem 11

Let a_n be defined by $a_0 = 678912$ and

$$a_{n+1} = 3a_n + 1 \text{ for } n = 1, 2, 3, \dots$$

Find the limit of the sequence $\{a_n\}$

Solution: A finite limit does not exist. To see this, iterate:

$$a_1 = 3a_0 + 1$$

$$a_2 = 3a_1 + 1 = 3^2a_0 + 3 + 1$$

$$a_3 = 3a_2 + 1 = 3^3a_0 + 3^2 + 3 + 1$$

These first few values show that $a_n \rightarrow \infty$