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Suppose that $f(x)$ is continuous at $x = c$ and $f(c) > 0$. Show that there is a $\delta > 0$ such that $f(x) > f(c)/2$ for $|x - c| < \delta$ and $x \in \text{Dom}(f)$.

Proof. Since f is continuous at c , given $\epsilon = f(c)/2$ there is a $\delta > 0$ such that

if $|x - c| < \delta$, $x \in \text{Dom}(f)$ then $|f(x) - f(c)| < \epsilon = f(c)/2$.

For such x

$$-f(c)/2 < f(x) - f(c) < f(c)/2$$

so

$$f(x) > f(c)/2.$$