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Prove that if a sequence converges it has exactly one limit point. Is the converse true?

Proof: First if $a_n \rightarrow c$ given an $\varepsilon > 0$ there is an $N(\varepsilon)$ such that

$$|a_n - c| < \varepsilon \text{ for } \underline{\text{all}} \ n \geq N(\varepsilon),$$

so c is the only limit point of the sequence.

The converse is not true. Set

$$\begin{aligned} a_{2n} &= 1 \\ a_{2n+1} &= 2n+1 \end{aligned}$$

The sequence $\{a_n\}$ has one limit point, but does not converge.