Page 34, Problem 7

Suppose that $a_n \to a$ and $a_n \geq b$ for each n. Prove that $a \geq b$.

Solution. Suppose that $a < b$. Then we can choose an $\varepsilon > 0$ so that

$$a + \varepsilon < b.$$

Next, since $a_n \to a$, there is a $N(\varepsilon)$ such that

$$a - \varepsilon < a_n < a + \varepsilon \quad \text{for all } n \geq N(\varepsilon).$$

Thus we have

$$a_n < a + \varepsilon < b,$$

which contradicts the assumption that $a_n \geq b$