Page 24, Problem 6

Prove that if n is a positive integer the $n^2 + 5n$ is divisible by 6.

The proof is by induction on n.

Since $1^2 + 5*1 = 6$, the assertion is true for n = 1.

The inductive step: Suppose that $n^2 + 5n$ is divisible by 6.

Next,

$$(n+1)^2 + 5(n+1)$$
$$=$$
$$n^3 + 3n^2 + 3n + 1 + 5n + 5$$
$$=$$
$$n^2 + 5n + 3n^2 + 3n + 6$$
$$=$$
$$(n^2 + 5n) + 3n(n+1) + 6$$

Then:

by the inductive hypothesis $(n^2 + 5n)$ is divisible by 6;

since n and n+1 are consecutive integers, one of them is even, so n(n + 1) is divisible by 2 and 3n(n + 1) is divisible by 6.

Consequently $(n + 1)^2 + 5(n+1)$ is divisible by 6.

This completes the inductive step