Page 24, Problem 3

Suppose that a, b, c, d are positive numbers such that a/b < c/d. Prove that

$$\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}.$$

Proof: Start with the first inequality

$$\frac{a}{b} < \frac{a+c}{b+d}$$

an work backwards. Since a, b, c, and d are all positive we can multiply the inequalities below without changing their sense:

$$\frac{a}{b} < \frac{a+c}{b+d}$$

$$\Leftrightarrow$$

$$a(b+d) < b(a+c)$$

$$\Leftrightarrow$$

$$ab + ad < ba + bc$$

$$\Leftrightarrow$$

$$ad < bc$$

$$\Leftrightarrow$$

$$\frac{a}{b} < \frac{c}{d}$$

Since the last inequality is true, we can reverse directions of the argument to establish the first.

The proof of the second inequality in the statement of the problem is similar