

Find an explicit one-to-one correspondence between the interval  $(-1, 7)$  and the set of all real numbers,

Solution: Let

$$f(x) = \frac{1}{7-x} - \frac{1}{x+1}$$

Then  $f(x)$  is a mapping of  $(-1, 7)$  to the reals:  $f(x) \rightarrow -\infty$  as  $x \rightarrow 1^+$  and  $f(x) \rightarrow +\infty$  as  $x \rightarrow 7^-$ .

Furthermore  $f(x)$  is one-to-one on  $(-1, 7)$  for its derivative is positive on this domain:

$$f'(x) = \frac{1}{(7-x)^2} + \frac{1}{(x+1)^2} > 0$$

The graph of  $f(x)$ :

