

Construct a function which gives an explicit correspondence between $N \times N$ and N , where N is the set of positive integers.

Solution: We have

<p>The couples:</p> <p>$(1,1) = c(1),$ $(1,2) = c(2), (2,1) = c(3),$ \cdot \cdot</p> <p>the couples (i, j) with $i+j = n$:</p> <p>$(1,n-1), (2,n-2), \dots (n-1,1)$</p> <p>---> ---> ---></p> <p>The couples (i, j) with $i + j = n+1$</p> <p>$(1,n) = c(N(n) + 1),$ $(2,n-1) = c(N(n) + 2),$ \cdot \cdot $(i,n+1-1) = c(N(n) + i),$ \cdot \cdot $(n, 1) = c(N(n) + n)$</p>	<p>Number of couples (i, j) with $i + j = k$:</p> <p>1 2 \cdot \cdot</p> <p>$n-1$ --- sum = $1 + 2 + \dots (n-1) = (n-1)n/2$</p> <p>Set $N(n) = (n-1)n/2$</p> <p><--- <--- <---</p>
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Thus the mapping of $N \times N$ to N is given by

$$(i, j) = c(N(n) + i) \rightarrow N(n) + i$$

where $i + j = n + 1, n = 1, 2, \dots$, and $N(n) = (n-1)n/2$.